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Longevity gap and pension contribution cap

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ABSTRACT

A basic function of public pension systems is to guarantee a satisfactory old-age income for short-sighted low earners. In proportional (i.e., earnings-related) systems, this requires a sufficiently high contribution rate. At the same time, there should be a cap on the pension contribution base to leave sufficient room for the efficient private savings of prudent high earners. Taking into account the dependence of life expectancy on the earnings (figuratively called longevity gap), a well-chosen cap has an additional advantage: it limits the unintended income redistribution from the short-lived to the long-lived. Our strongly stylized model is able to illustrate numerically the impact of the contribution rate and of the cap on the social welfare and the unintended income redistribution.

JEL codes: D10, H55, I38

Keywords: public pension system, cap, longevity gap, income redistribution

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Élettartamrés és nyugdíjjárulék plafon

SIMONOVITS ANDRÁS

ÖSSZEFOGLALÓ

A tb-nyugdíjrendszerek egyik legfontosabb feladata, hogy kielégítő időskori jövedelmet nyújtson a kiskeresetű rövidlátó dolgozóknak. Keresetarányos rendszerekben ez elegendően magas járulékkulcsot követel meg. Ugyanakkor a járulékalaphoz plafont kell rendelni, hogy a nagykeresetű és előrelátó dolgozók hatékony magánmegtakarításának elegendő tér maradjon. Figyelembe véve, hogy a várható élettartam függ a keresettől (képletesen: létezik az élettartamrés), a jól megválasztott plafon további előnnyel jár: korlátozza a szándékolatlan jövedelem-újraelosztást a rövid életűektől a hosszabb életűek felé. Erősen stilizált modellünk képes számszerűen szemléltetni a járulékkulcs és a plafon hatását a társadalmi jólétre és a szándékolatlan újraelosztásra.

JEL: D10, H55, I38

Kulcsszavak: tb-nyugdíjrendszer, plafon, élettartamrés, jövedelem-újraelosztás

Longevity gap and pension contribution cap

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Abstract

A basic function of public pension systems is to guarantee a satisfactory old-age income for short-sighted low earners. In proportional (i.e. earnings-related) systems, this requires a sufficiently high contribution rate. At the same time, there should be a cap on the pension contribution base to leave sufficient room for the efficient private savings of prudent high earners. Taking into account the dependence of life expectancy on the earnings (figuratively called longevity gap), a well-chosen cap has an additional advantage: it limits the unintended income redistribution from the short-lived to the long-lived. Our strongly stylized model is able to illustrate numerically the impact of the contribution rate and of the cap on the social welfare and the unintended income redistribution.

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1 Introduction

At a first approximation, the contribution rate is the most important parameter of any balanced public pension system. Going deeper, however, the *cap on the contribution base* or shortly, the cap or equivalently, the maximum taxable earnings (MTE) is also important. If the cap is low, then the effective contribution rate is also low. In addition, the progressivity of the pension system is also important: the more progressive the system, the lower the balanced contribution rate. A basic difference between capped and progressive systems is the following: in the former, the worker does not contribute above the cap and does not receive any benefit above the implied benefit cap; in the latter, only the benefit is reduced above a threshold but the contribution is uncapped.

While the literature on the contribution rate is large, that on the cap is quite small. The most important sources are as follows: Diamond–Orszag (2004, pp. 65–66) depicted how much changed the share of earnings above the cap in the US: from 16% (1977) to 10% (1980), then stabilized at the initial value; and the share of the earners above the cap: sinking from 15% to 6% by 1983 and then stayed there. Barr and Diamond (2008 p. 63) explained that diminishing the cap limits the coercion/compulsion of the higher-earners but may reduce the old-age consumption of lower-earners. Table 1 in Valdés-Prieto and Schwarzhaupt (2011) presents MTE data of 60 countries. OECD (2015, p. 129, Table 5.6) displays data on the more relevant cap-to-gross wage-ratio: it is around 1.2 in Sweden, 1.8 in Germany and 2.5 in the U.S.

It was Feldstein (1985) who first studied the socially optimal value of the contribution rate in a highly stylized quasi-stationary OLG model with wage and population growth. The basic idea is as follows: if there were no mandatory pension, the representative worker would save privately and at retirement, she would drawdown on her accumulated savings. Due to strong discounting of the future, the old-age consumption, however, would be too low. To make up the saving gap, a paternalistic government mandates a public pension: the worker pays pension contribution and at retirement receives public pension. To choose the optimal contribution rate, the government raises the individual discount factor in calculating the social welfare. Due to various controversial assumptions, Feldstein obtained quite low (sometimes zero) socially optimal contribution rates. Revising that model, Andersen and Bhattacharya (2011), Simonovits (2018, Appendix D) have obtained a more realistic picture (see also Figure 1 below).

It were Valdés-Prieto and Schwarzhaupt (2011) who worked out the theory of optimal compulsion. From the rich menu, we shall confine our attention to their Section 6 on the cap. Relying on an observation that higher paid workers are ready to save proportionally more than others, the foregoing authors heuristically determined the socially optimal value of the cap, being around the 80th percentile. Having streamlining and generalizing Feldstein (1985) into a multi-type model, where the discount factor is increasing in wage, and incorporating Valdés-Prieto and Schwarzhaupt (2011) ideas, Simonovits (2015) and (2018, Chapters 4 and 7) optimized the contribution rate and the cap together in a proportional (i.e., earning-related) systems.

We reconsider the problem and our results depend on the parameters, especially on the elasticity of the discount coefficient with respect to the wage. In our parameter interval, the optimal contribution rate hardly differs from the maximal rate, equalizing the young- and old-age consumption. On the other hand, the relative efficiency of the system is quite insensitive to the cap in a wide interval, typically not containing either the minimum or the maximum wage. (Note, however, that in Simonovits (2018c, Section 3.3) and our

Example 3 below, where there exists only two types, the optimal cap is equal to the minimum.) Lee et al. (2021) considered a similar model to ours but they added flexible labor supply and neglected private savings. They only proved that at least a very high cap is socially better than no cap.

To obtain a fuller picture, note that already Liebmann (2002) and Whitehouse and Zaidi (2008) underlined the impact of socioeconomic differences in mortality on income redistribution in public pensions. Recently a lot of researchers discussed the widening longevity gap (i.e., the dependence of life expectancy on lifetime incomes) and its impact on the unintended redistribution from the shorter-lived lower paid to the longer-lived, higher paid (e.g., Chetty et al., 2016 and Holzmann et al., 2020). Most analysts (e.g., Ayuso et al, 2017; Haan et al., 2020; Simonovits, 2021a; Simonovits and Lackó, 2022) suggested the strengthening the progressivity of the benefit rules: i.e., faster reducing the marginal accrual rate in calculating the pension benefits, notwithstanding the possible repercussion on labor supply.

To our knowledge, the present paper is the first one that studies the interaction of cap and longevity gap in a proportional (DC) public pension model (though both categories appear in Sheshinski and Coliendo, 2020; where the focus is on progressivity). Generalizing Simonovits (2015), in the present paper we show by numerical calculations that by introducing the longevity gap, the socially optimal cap becomes lower: furthermore, the cap reduces the unintended income redistribution caused by the longevity gap. We borrow a stringent assumption of other related models: the life expectancy only depends on the gross wage but is independent of pension and private saving.

We consider a time-invariant model with constant parameter values. In reality, many parameters are time-variant, and modeling any pension reform requires a dynamic model (e.g., Fehr et al., 2013 discussing the optimal introduction of progressivity in the German public pension system). For example, if we wanted to model the impact of the Hungarian government's elimination of the cap in 2013 or its eventual restoration in an otherwise almost proportional system (opening and closing room for unintended redistribution), we would need a dynamic model etc. The same applies to a fourth characteristic, the interaction of indexation rules and longevity gap (cf. Simonovits, 2021b).

The structure of the remainder of the paper is as follows. Section 2 presents the analytical model and Section 3 displays the numerical calculations. Section 4 draws the conclusions, while an Appendix reports some sensitivity analysis.

2 Analytical model

In this Section we introduce our analytical model in two parts: no cap and cap. To reduce the number of parameters, following Simonovits (2015) we hide the growth of real wages and of population into the relative interest factor, namely the ratio of the interest factor to the growth factor. We keep neglecting the population change but take into account a basic fact: individuals spend much shorter time in retirement than in work.

2.1 No cap

Assume that every worker starts to work and retires at the same ages, she is characterized by her total wage compensation w, she pays pension contribution τw for a period of unit length and receives a wage-dependent per-period benefit $b(w)$ for a period of length to be called duration, $m(w)$, $0 < m(w) < 1$, i.e. a total $m(w)b(w)$, where $b(\cdot)$ and $m(\cdot)$ are weakly and strictly increasing functions, respectively. Considering only public pensions, we assume unisex populations. We assume a general cumulated distribution function $F(w)$ and denote its normalized expected value by $\mathbf{E}w = 1$. If there is no misunderstanding, we drop the wage-dependence. We shall denote the average duration by $\mu = \mathbf{E}m$. To avoid absurd outcomes, we shall make the following additional realistic assumption: $m(\cdot)$ is (strictly) concave. By Jensen-inequality, typically $\mu < m(1)$. In a traditional framework, a worker annually earns w for S years and receives annual benefit b for T years. Dividing by T, we obtain $m = T/S$.

We shall start with a scheme where every benefit is *proportional* to the corresponding contributions or equivalently, to gross wages:

$$b = \beta w,$$

where $\beta > 0$ is the accrual ratio.

To measure (expected) lifetime redistribution, we shall consider the *lifetime balance* of a worker with wage w:

$$z = \tau w - \beta m w. \tag{1}$$

We shall assume that the system is *balanced*, i.e., the *expected* balance is equal to zero. Using (1) and $\mathbf{E}w = 1$:

$$\mathbf{E}z = \tau - \beta \mathbf{E}(mw) = 0,$$

establishing proportionality between the contribution rate τ and the accrual rate β:

$$\beta = \frac{\tau}{\mathbf{E}(mw)}. \tag{2}$$

Substituting (2) into (1) yields

$$z = \tau w \left\{ 1 - \frac{m}{\mathbf{E}(mw)} \right\}.$$

Since $m(w)$ is increasing, $\mathbf{E}(mw) > \mu$; moreover, $\beta < \tau/\mu$. Furthermore, considering continuous distribution functions, denote the critical wage w°, where $z(w^{\circ}) = 0$. Then $z(w) > 0$ for $w < w^{\circ}$ (loser) and $z(w) < 0$ for $w > w^{\circ}$ (gainer).

2.2 Cap

Having finished our preparation, next we introduce the minimum wage w_{m} and the *cap* on the contribution base $\bar{w}$. Earning below and at the cap is fully accounted in both the contributions and the benefits; the part of earning above the cap is neglected on both sides. To avoid unnecessary complications, we assume that $\bar{w} \geq w_{\mathrm{m}} > 0$. (Otherwise by multiplying the contribution rate by $\bar{w}/w_{\mathrm{m}} < 1$, the effective cap would rise to w_{m} and everybody would pay the same contribution $\tau\bar{w}$.)

We define the covered wage $\tilde{w} = \min[w, \bar{w}]$, the benefit $\tilde{b} = \beta\tilde{w}$ and the lifetime balance $\tilde{z} = (\tau - m\beta)\tilde{w}$.

Taking expectations: $\mathbf{E}\tilde{z} = 0$. Separating the two domains, and using notation $\mathbf{E}_{x \leq \bar{x}}\, x$ for expectation of a random variable x, and $\mathbf{P}(x > \bar{x})$ for probability of the corresponding event, the balance condition now reduces to

$$\mathbf{E}_{w \leq \bar{w}}[(\tau - m\beta)w] + \bar{w}\mathbf{P}(w > \bar{w})(\tau - m\beta) = 0,$$

or after rearrangement,

$$\tau\left[\mathbf{E}_{w\le\bar{w}}\, w+\bar{w}\mathbf{P}(w>\bar{w})\right]=\beta\left[\mathbf{E}_{w\le\bar{w}}\,(mw)+\bar{w}\mathbf{E}_{w>\bar{w}}\, m\right]. \tag{3}$$

Proportionality between τ and β has been again established and β can be expressed.

It is worth recalling the simultaneous impact of the contribution rate and the cap: the higher the contribution or the cap, the greater the pension system. For example, the *effective contribution rate*—defined as $\tau\min[1,\bar{w}/w]$—is a proportional function of the original contribution rate and a weakly increasing function of the cap.

2.3 Social welfare

Having introduced the core of the model, we turn now to social welfare. First we have to introduce young- and old-age consumption, lifetime utility and social welfare functions. Without private savings, the consumption pairs would be equal to

$$c_0=w-\tau\tilde{w} \qquad \text{and} \qquad d_0=\beta\tilde{w}.$$

We also add wage-dependent nonnegative *private saving* $s\ge 0$ to ensure some old-age consumption if the contribution rate or the cap is too low. Following Yaari (1965), we assume that everybody can buy a perfect life annuity converting her accumulated savings Rs (R being the compounded interest factor) by multiplying with a factor m^{-1}:

$$c=c_0-s \qquad \text{and} \qquad d=d_0+m^{-1}Rs. \tag{4}$$

We shall use a very simple *discounted* lifetime utility function:

$$U(c,d)=\log c+\delta m\log d. \tag{5}$$

where $\delta\in[0,1]$ stands for the wage-dependent discount factor. We assume that the higher the wage, the weaker the discounting (see Valdés-Prieto and Schwarzhaupt, 2011).

We shall now determine the optimal private savings. Inserting (4) into (5):

$$U[s]=\log(c_0-s)+\delta m\log(d_0+m^{-1}Rs). \tag{6}$$

Take the derivative of (6):

$$U'[s]=\frac{-1}{c_0-s}+\frac{\delta R}{d_0+m^{-1}Rs}=0$$

and after rearrangement:

$$d_0R^{-1}+m^{-1}s=\delta c_0-\delta s.$$

Solving for s and replacing the possible negative solution by 0 yields

$$s^{\mathrm{o}}=\frac{[\delta c_0-d_0R^{-1}]_+}{\delta+m^{-1}},$$

where x_+ is the positive part of a real x.

One reason for introducing mandatory public pensions is to force shortsighted workers to save more than they would voluntarily. Technically, we adopt the approach introduced by Feldstein (1985): overwriting the shortsightedness of workers, the paternalistic government does not discount old-age consumption in calculating the social welfare function.

We do not follow, however, Feldstein's allowance of negative saving and underestimation of future benefits, implying undervaluation of the public pension system. (see Simonovits, 2018, Section 3.2 and Appendix D). Substituting $s^{\rm o}$ into (6) and eliminating the discounting, yields the paternalistic, *undiscounted* indirect utility function

$$\mathbf{U} = \log(c_0 - s^{\rm o}) + m\log(d_0 + m^{-1}Rs^{\rm o}).$$

We define the social welfare function as the corresponding expected utility function

$$V_0(\tau, \bar{w}) = \mathbf{E}\mathbf{U}.$$

With the help of this function we can determine the socially optimal contribution rate and the cap but the dependence of the welfare on the nonoptimal pairs remains in the shadow. Therefore we introduce the *relative efficiency* (or consumption equivalent variation) of the capped pension system in terms of the no-pension system 0, denoted by ε. We have the following implicit definition: this is the number by which multiplying the wages in the no-pension system, the resulting social welfare of the no-pension system equals to that of the capped pension system with the original wages:

$$V(0, 0, \varepsilon) = V(\tau, \bar{w}, 1).$$

Note that the wage-dependent discount factors and durations are *not* modified.

Substituting ε into the logarithmic utility functions,

$$V(0, 0, \varepsilon) = V(0, 0, 1) + (1 + \mu)\log\varepsilon.$$

Hence we can return to V_0:

$$\log\varepsilon = \frac{V_0(\tau, \bar{w}) - V_0(0, 0)}{1 + \mu}, \qquad \text{i.e.,} \qquad \varepsilon = \exp\left[\frac{V_0(\tau, \bar{w}) - V_0(0, 0)}{1 + \mu)}\right].$$

It is easy to understand that the lower the contribution rate, the higher the conditionally optimal cap. The really difficult question is: what is the socially optimal pair for which the lower-earners receive sufficiently high replacement and for the higher-earners remain sufficient room for private savings?

It is time to touch another side-effect of the longevity gap: even in an ex ante proportional system, there is an *unintended income redistribution*, to be measured by the standard deviation of the net annual balances per year: its squared value is $\mathbf{D}^2 z = \mathbf{E}z^2$ (a similar indicator was used by Holzmann et al. (2020); Sheshinski and Coliendo (2021). It is a simple measure but does not differentiate between low-earners supporting high-earners (perverse redistribution) or vice versa.

2.4 Four examples

Here we present three simple examples with one or two representative workers and a fourth one with an arbitrary number of types.

Example 1. Having a representative agent, with $w = 1$, no effective cap: $\bar{w} \geq 1$, and no saving, the social welfare function is given by

$$V[\tau] = \log(1 - \tau) + \mu\log(\mu^{-1}\tau).$$

Taking its derivative and setting the derivative to zero, the optimal contribution rate can be determined:

$$0 = V'[\tau] = \frac{1}{1-\tau} + \frac{\mu}{\tau} \Rightarrow \bar{\tau} = \frac{\mu}{1+\mu}. \tag{7}$$

The corresponding consumption pair are

$$\bar{c} = \frac{1}{1+\mu} \qquad \text{and} \qquad \bar{d} = \frac{1}{1+\mu}.$$

We could refer to $\bar{\tau}$ as the maximal contribution rate, because for even higher contribution rate, the resulting young-age consumption would be lower than the old-age one.

Numerically, $\mu = 0.5$ yields $\bar{\tau} = 1/3$, $\bar{c} = 2/3 = \bar{d}$. Of course, this value is too high but by introducing reduced family size, value of leisure and other real life complications, it can diminished.

Example 2. (Simonovits, 2018, Section 3.2.) Retaining the representative agent of Example 1 but adding private saving, the picture becomes more complex. Without going into the details, we recall that for any interest factor R, there is a critical discount factor δ_R such that the social welfare of no pension is equal to that of the maximal pension: $V(0) = V(\bar{\tau})$. We shall consider subcritical discount factors $0 < \delta < \delta_R$, where there is a critical contribution rate $0 < \tau_{\mathrm{o}} < \bar{\tau}$ such that $V(0) = V(\tau_{\mathrm{o}})$ and in the interval $\tau_{\mathrm{o}} < \tau < \bar{\tau}$, the public pension system delivers higher welfare than the no pension (see also Figure 1 below).

Example 3. (Generalization of Simonovits, 2018c, Section 4.4.) In the simplest heterogeneous model, there exist two workers with earnings w_{m}, w_{M}; frequencies f_{m}, f_{M}; and discount factors $\delta_{\mathrm{m}} = 0 < 1 = \delta_{\mathrm{M}}$. If the contribution rate is sufficiently high, then the socially optimal cap is equal to the minimum wage. The calculation becomes more complex when there is a longevity gap: $m_{\mathrm{m}} < m_{\mathrm{M}}$. For example, (3) reduces to

$$\tau(f_{\mathrm{m}} w_{\mathrm{m}} + f_{\mathrm{M}} \bar{w}) = \beta(f_{\mathrm{m}} m_{\mathrm{m}} w_{\mathrm{m}} + f_{\mathrm{M}} m_{\mathrm{M}} \bar{w}),$$

(1) to

$$z_{\mathrm{m}} = (\tau - \beta m_{\mathrm{m}}) w_{\mathrm{m}} > 0 \qquad \text{and} \qquad z_{\mathrm{M}} = (\tau - \beta m_{\mathrm{M}}) \bar{w} < 0.$$

There is a single obvious exception where there are many types but the cap does not help.

Example 4. For any number of types, if the discount coefficient δ is independent of the wage, and $\delta R < 1$, than both the optimal contribution rate and the cap are maximal.

The following qualitative issues should be discussed: What is the range of optimal contribution rates and caps under the longevity gap? How can unintended income redistribution be reduced by appropriately choosing the contribution rate and the cap? Our simulations will demonstrate that the optimal contribution rate is quite close to the maximum, determined in (7) of Example 1 and the corresponding optimal cap is equally far from the minimum and the maximum wages.

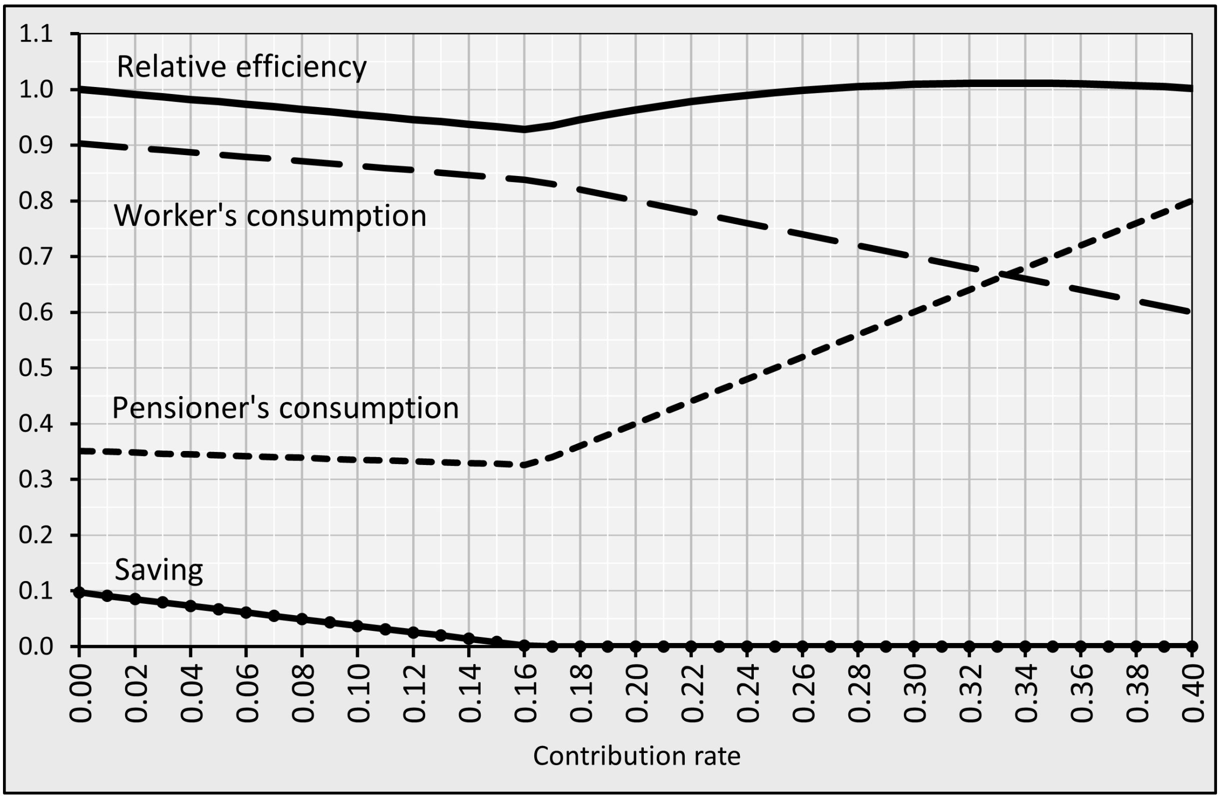
3 Numerical illustrations

Our formulas are so complex that we are unable to make definite deductions, even in Example 3. Rather, we experiment with numerical illustrations, lacking reliable calibration. First of all, to obtain meaningful numbers, we have to connect the per-period discount and interest coefficients with their annual versions. The compound interest factor is a power function of the annual discount factor $R = R[1]^A$ and similarly, $\delta = \delta[1, w]^A$, neglecting the dependence of half the adult lifespan A on w.

3.1 No longevity gap

We start with Example 2, for example, assuming $\delta[1] = 0.95$ and $R[1] = 1.02$, Figure 1 displays how the mandatory contribution crowds out the efficient private saving, reaching the minimum efficiency of 0.93 at $\tau_{\mathrm{m}} = 0.16$; the original efficiency is recovered at $\tau_{\mathrm{o}} = 0.27$, while the maximum efficiency of 1.01 materializes at $\bar{\tau} = 0.33$...

Figure 1. The impact of the contribution rate on welfare



To model wage heterogeneity, we shall use the Pareto-distribution of parameter σ defined by

$$F(w) = 1 - (w_{\mathrm{m}}/w)^{\sigma}, \qquad \text{where} \qquad w \geq w_{\mathrm{m}} > 0.$$

The corresponding density function is given by

$$f(w) = \sigma w_{\mathrm{m}}^{\sigma}/w^{1+\sigma}, \qquad \text{where} \qquad w \geq w_{\mathrm{m}} > 0.$$

Normalizing the wages, we assume that $\mathbf{E}w = 1$, i.e., $w_{\mathrm{m}} = (\sigma - 1)/\sigma$. For $\sigma = 2$, the minimum wage is half the average: $w_{\mathrm{m}} = 1/2$. The first row of Table 1 displays that the share of fully covered workers. For example, for $\bar{w} = 1.5$, $\mathbf{P}(\bar{w}) = 0.889$.

Here we can determine the covered wages. For the Pareto-distribution, the two terms are as follows:

$$\omega_1 = \int_{w_{\rm m}}^{\bar{w}} w\sigma w_{\rm m}^{\sigma} w^{-1-\sigma}\, dw = \frac{\sigma w_{\rm m}^{\sigma}}{\sigma-1}\left[\frac{1}{w_{\rm m}^{\sigma-1}} - \frac{1}{\bar{w}^{\sigma-1}}\right] = 1 - \frac{w_{\rm m}^{\sigma-1}}{\bar{w}^{\sigma-1}}$$

and

$$\omega_2 = \bar{w}\frac{w_{\rm m}^{\sigma}}{\bar{w}^{\sigma}} = \frac{w_{\rm m}^{\sigma}}{\bar{w}^{\sigma-1}}.$$

The second row in Table 1 below shows this dependence for $\sigma = 2$, $\bar{w} = 1.5$, the share of earnings is equal to 0.667.

Table 1. The shares of fully covered workers and of wages

Cap	0.5	1.0	1.5	2.0	3.0	4.5
Share of fully covered workers	0	0.75	0.889	0.938	0.972	0.988
Share of covered wages	0.5	0.75	0.833	0.875	0.917	0.944

In practice, we shall work with $n = 20$ income classes of equal probability, with divisor points W_i:

$$1 - (w_{\rm m}/W_i)^{\sigma} = i/n, \qquad \text{i.e.,} \qquad W_i = w_{\rm m}[i/(n-i)]^{1/\sigma}.$$

The class income is represented by

$$w_i = \sqrt{W_i W_{i+1}}, \qquad i = 1, \ldots, n-1.$$

yielding $w_{\rm M} = 4.472$.

We parameterize the wage dependence of the annual discount factor $\delta[1, w]$:

$$\delta[1, w] = \delta_{\rm M}[1] + (\delta_{\rm m}[1] - \delta_{\rm M}[1])e^{\eta(w_{\rm m}-w)}, \qquad w \geq w_{\rm m}, \qquad \delta_{\rm m}[1] < \delta_{\rm M}[1] \leq 1.$$

Obviously, $\delta[1, w]$ is increasing in w from $\delta[1, w_{\rm m}] = \delta_{\rm m}[1]$ to $\delta[1, \infty] = \delta_{\rm M}[1]$, and for simplicity, we shall refer to it η as elasticity of discounting with respect to wage. The higher the elasticity, the stronger the wage's impact on discounting.

We have four free parameters together but we reduce them to one by assuming that

$$R[1] = 1.02, \qquad \text{and} \qquad \delta_{\rm m}[1] = 0.95, \qquad \delta_{\rm M}[1] = 1.$$

In Table 2, we shall change the elasticity of discount factor between 0 and 0.4. Note that in our grid, the socially optimal contribution rate is always (near) 0.33 (see the Appendix), therefore we fix that value. We evaluate the relative efficiency of the capped system for various caps, starting with the minimal wage 0.5 and ending with the maximal wage 4.5. For low elasticities, the cap diminishes the efficiency (in the spirit of Example 4) but at higher elasticities, the cap has benefits.

For example, in the first row of Table 2, any cap reduces the efficiency from 1.116 (the optimum); in the last row of Table 2, the relative efficiency of the uncapped system is slightly lower than the optimally capped one's (italicized) between 1.5 and 2 times the average wage: $1.065 < 1.068$.

Table 2. The impact of the elasticity parameters on the caps' efficiency: no gap

Sensitivity of discount rate η	Relative efficiency of cap at multiples of average wage 0.5	1	1.5	2	3	4.5
0.0	1.065	1.103	1.108	1.110	1.114	*1.116*
0.1	1.058	1.091	1.096	1.096	1.096	*1.098*
0.2	1.052	1.081	*1.086*	*1.086*	1.085	1.085
0.3	1.047	1.073	*1.076*	*1.076*	1.075	1.074
0.4	1.043	1.066	*1.068*	*1.068*	1.067	1.065

3.2 Longevity gap

We turn now to the numerical analysis of the impact of the longevity gap. Concerning survival probabilities, we rely on a simple power function of the relative age a (cf. Sheshinski and Coliendo, 2020):

$$p_a(w) = 1 - a^{\gamma+\psi w}, \qquad 0 < a < 1, \qquad \gamma, \psi > 0.$$

Note that p_a is decreasing with a, starting at 1 and ending at 0 but increasing with w in between.

With simple calculation, the duration–wage-function is given by

$$m(w) = \int_0^1 [1 - a^{\gamma+\psi w}]\, da = 1 - \frac{1}{1+\gamma+\psi w} = \frac{\gamma+\psi w}{1+\gamma+\psi w}.$$

We shall choose $\gamma = 0.77$ and $\psi = 0.02$, where $\mu = 0.5$, the relative gap $\Delta m = m[w_{0.9}] - m[w_{0.1}] = 0.156$ (with 90th and 10th percentile wages).

What is the impact of the longevity gap? The socially optimal contribution rate remains (near) 0.33. Even for the lowest elasticities, the cap (around 3) slightly raises the efficiency but at higher elasticities, the cap (between 2 and 3 times the average wage) has real benefits. In the first row of Table 3, 1.083 > 1.079; in the last row of Table 3, the relative efficiency of the optimal uncapped system is significantly lower than the optimally capped one's near the average wage: 1.025 < 1.045.

Table 3. The impact of the elasticity parameters on the caps' efficiency: gap

Sensitivity of discount rate η	Relative efficiency of cap at multiples of average wage 0.5	1	1.5	2	3	4.5
0.0	1.051	1.083	1.084	1.082	*1.083*	1.079
0.1	1.044	1.070	*1.071*	1.068	1.063	1.059
0.2	1.038	1.060	*1.061*	1.058	1.051	1.045
0.3	1.033	*1.052*	1.051	1.048	1.042	1.034
0.4	1.029	*1.045*	1.043	1.040	1.033	1.025

Taking into account the unintended redistribution, the case for a lower cap is even stronger (Table 4).

Table 4. Standard deviation of balances as a function of cap

Standard deviation of balances at multiples of average wage					
0.5	1	1.5	2	3	4.5
0.022	0.037	0.049	0.060	0.079	0.103

Perhaps the best illustration of these numbers is to compare them with a two-type case in Example 3. Let us have $f_{\rm m} = 0.5$; $w_{\rm m} = 0.5$, implying $w_{\rm M} = 1.5$ and the capless variance is equal to

$$\mathbf{D}^2 z = f_{\rm m}(\tau - \beta m_{\rm m})^2 w_{\rm m}^2 + f_{\rm M}(\tau - \beta m_{\rm M})^2 w_{\rm M}^2, \qquad \text{where} \qquad \beta = \frac{f_{\rm m} w_{\rm m} + f_{\rm M} w_{\rm M}}{f_{\rm m} m_{\rm m} w_{\rm m} + f_{\rm M} m_{\rm M} w_{\rm M}} \tau.$$

Then $\mathbf{D} z = 0.115$; close to the last entry 0.103.

4 Conclusions

Introducing the longevity gap into our model with contribution rate and cap, we have extended the analysis to a domain where even the proportional pension system causes unintended redistribution. Several experimental calculations show the presence of the longevity gap hardly influences the optimal value of the contribution rate but pushes the optimal interval of the cap to the left. Of course, we have only considered the simplest analytical model in a constrained interval of some parameters. We only hope that the basic insight remains true in a wider set of the parameters (see Appendix), and possibly in a wider world of models. Further research is needed to corroborate our initial results. For example, what would happen if a flat pillar had been added to the proportional one. Or if the workers underestimated their own life expectancies. It would be much more difficult to model a dynamic system, where the cap is reintroduced, like it should be in the Hungarian pension system.

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Appendix: Sensitivity analysis

This Appendix contains some sensitivity analysis to Tables 2 and 3, fixing $\eta = 0.4$, making the annual interest rate rising with the wage. We shall use a simple function

$$R[1,w] = R_{\mathrm{M}}[1] + (R_{\mathrm{m}}[1] - R_{\mathrm{M}}[1])e^{\xi(w_{\mathrm{m}}-w)}, \qquad w \ge w_{\mathrm{m}}, \qquad R_{\mathrm{m}}[1] < R_{\mathrm{M}}[1].$$

Obviously, $R[1,w]$ is increasing in w from $R[1,w_{\mathrm{m}}] = R_{\mathrm{m}}[1] = 1$ to $R[1,\infty] = R_{\mathrm{M}}[1] = 1.02$, and for simplicity, we shall refer to its ξ as elasticity of interest factor with respect to wage. The higher the elasticity, the stronger the wage's impact on the interest factor. The per-period interest factor is given by $R(w) = R[1,w]^A$.

Moreover, we vary the contribution rate from 0.3 to 0.36 through 0.33. Even if there is no gap, in all the three cases, the middle value is the optimum, while the optimal cap diminishes with rising elasticity.

Table A.1. The impact of the contribution rate on the caps' efficiency: no gap

Sensitivity of interest rate ξ	Contribution rate τ	Relative efficiency of cap at multiples of average wage 0.5	1	1.5	2	3	4.5
0.0	0.30	1.124	1.166	1.172	1.173	1.173	1.174
	0.33	1.134	1.170	1.176	1.176	1.176	*1.177*
	0.36	1.143	1.170	1.174	1.174	1.174	1.175
0.2	0.30	1.115	1.153	1.157	1.158	1.157	1.155
	0.33	1.125	1.157	1.160	*1.161*	1.159	1.157
	0.36	1.132	1.156	1.159	1.159	1.157	1.155
0.4	0.30	1.107	1.143	1.145	1.146	1.144	1.142
	0.33	1.117	1.146	*1.148*	*1.148*	1.146	1.144
	0.36	1.124	1.145	1.147	1.146	1.144	1.141

$\eta = 0.4$.

In Table A.2, we add longevity gap, and observe that the middle contribution rate remains the optimum, and the optimal cap is lower than without gap.

Table A.2. The impact of the contribution rate on the caps' efficiency: gap

Sensitivity of interest rate ξ	Contribution rate τ	Relative efficiency of cap at multiples of average wage 0.5	1	1.5	2	3	4.5
0.0	0.30	1.106	1.141	1.143	1.141	1.136	1.129
	0.33	1.115	1.144	*1.146*	1.143	1.138	1.132
	0.36	1.123	1.144	1.144	1.141	1.136	1.130
0.2	0.30	1.097	1.128	1.127	1.124	1.117	1.108
	0.33	1.106	*1.130*	*1.130*	1.126	1.119	1.110
	0.36	1.112	1.129	1.128	1.124	1.117	1.108
0.4	0.30	1.089	1.117	1.115	1.112	1.104	1.094
	0.33	1.098	*1.119*	1.117	1.113	1.106	1.096
	0.36	1.103	1.118	1.115	1.111	1.103	1.093

$\eta = 0.4$.