INSTITUTE OF ECONOMICS
Centre for Economic and Regional Studies

Közgazdaság- és Regionális Tudományi Kutatóközpont
KÖZGAZDASÁG-TUDOMÁNYI INTÉZET

KRTK-KTI WORKING PAPERS | KRTK-KTI MŰHELYTANULMÁNYOK

Financial crisis and inequality in Hungary

ISTVÁN KÓNYA

August 2021

KRTK-KTI WP – 2021/32

https://kti.krtk.hu/wp-content/uploads/2021/08/KRTKKTIWP202132.pdf

INSTITUTE OF ECONOMICS, CENTRE FOR ECONOMIC AND REGIONAL STUDIES,
BUDAPEST, 2021

ABSTRACT

The goal of this paper is to analyze the connections Hungarian income and wealth distribution on the one hand, and the macroeconomics impacts of the global financial crisis of 2007-2008 on the other hand. To do this, I build a heterogenous agent, dynamic, general equilibrium model, which I calibrate using Hungarian income distribution data before the crisis. The model is then used to study both the impact of the financial crisis on income and wealth inequality, and the role of income and wealth inequality in the macroeconomic developments after the crisis. Results indicate that (i) the long-run capital stock rises, and the interest rate falls, but the effect is quantitatively small; (ii) the long-run income and wealth distributions only change moderately; and (iii) the short-run consumption response of low-wealth household is very strong, and drives a sizable aggregate consumption drop as well.

JEL codes: C63, E21, E44, E47

Keywords: heterogeneity, income shocks, simulations, borrowing constraint

István Kónya
Corvinus University, Budapest
and
Centre for Economic and Regional Studies, Budapest
e-mail: konya.istvan@krtk.hu

Pénzügyi válság és egyenlőtlenség Magyarországon

KÓNYA ISTVÁN

ÖSSZEFOGLALÓ

A cikk célja a magyar gazdaság jövedelem eloszlása, illetve a 2008-2009-es pénzügyi válság makroökonómiai hatásai közötti kapcsolat vizsgálata. Ehhez egy heterogén szereplős, dinamikus, általános egyensúlyi modellt építünk, amit magyar adatok segítségével számszerűsítünk. A modell segítségével vizsgálni tudjuk mind a pénzügyi válság hatásait a jövedelemegyenlőtlenségekre, mind a jövedelem egyenlőtlenség szerepét a válság makroökonómiai lefolyásában. A fő eredmények a következők: (i) a tőkeállomány hosszú távon nagyobb, a kamatláb pedig kisebb lesz, de a változás mértéke csekély; (ii) a hosszú távú jövedelem- és vagyoneloszlás kis mértékben módosul; (iii) az alacsony jövedelmű háztartások fogysztása rövid távon jelentősen csökken, és ez az aggregált fogyasztás érdemi esését is maga után vonja.

JEL: C63, E21, E44, E47

Kulcsszavak: heterogenitás, jövedelem sokkok, szimulációk, hitelfelvételi korlát

Financial crisis and inequality in Hungary*

István Kónya†

Abstract

The goal of this paper is to analyze the connections Hungarian income and wealth distribution on the one hand, and the macroeconomics impacts of the global financial crisis of 2007-2008 on the other hand. To do this, I build a heterogenous agent, dynamic, general equilibrium model, which I calibrate using Hungarian income distribution data before the crisis. The model is then used to study both the impact of the financial crisis on income and wealth inequality, and the role of income and wealth inequality in the macroeconomic developments after the crisis. Results indicate that (i) the long-run capital stock rises, and the interest rate falls, but the effect is quantitatively small; (ii) the long-run income and wealth distributions only change moderately; and (iii) the short-run consumption response of low-wealth household is very strong, and drives a sizable aggregate consumption drop as well.

1 Introduction

Allowing for household income heterogeneity is becoming more and more common in macroeconomic modeling. Since the seminal work of Aiyagari (1994) many articles have explored various extensions of the representative agent framework assumed in forward-looking, optimizing macroeconomic models.[1] Originally, the literature wanted to understand the effects of household heterogeneity on macroeconomic outcomes. It is easy to see that if financial markets are complete, (income) heterogeneity on its own has no effect on aggregate variables. If households can fully insure against idiosyncratic income shocks, the model behaves as if there was a single, representative household.

*This research was supported by the Higher Education Institutional Excellence Program 2020 of the Ministry of Innovation and Technology in the framework of the 'Financial and Public Services' research project (TKP2020-IKA-02) at Corvinus University of Budapest.

†Corvinus University and Centre for Economic and Regional Studies

[1] Additional important examples include Huggett (1997) and Marcet, Obiols-Homs and Weil (2007). Kaplan and Violante (2018) presents a broad recent overview of the literature.

Aiyagari (1994) and the subsequent literature therefore assumes that financial markets are incomplete. A typical modeling device is to assume an exogenous *borrowing constraint*, below which household net worth cannot fall. This implies that household behavior includes a precautionary saving motive. A standard consequence of such behavior is that in models with heterogenous households and incomplete financial markets, the aggregate capital stock is higher, and the equilibrium interest rate is lower than in the representative agent framework. While the literature mostly worked with real models in the past, a recent development is the appearance of so-called HANK (*Heterogenous Agent New Keynesian*) models (Kaplan, Moll and Violante, 2018), which try to integrate household heterogeneity into the transmission mechanism of monetary policy.

This paper uses the original Aiyagari (1994) approach to study income and wealth heterogeneity in Hungary. The main research question is to see how incorporating household heterogeneity the impact mechanism of the global financial crisis of 2008-2009. To answer this question I implement the shock caused by the financial crisis in such a way that requires household heterogeneity to be present. I do not aim for a full description of crisis impact, these were already studied in representative agent models by Benczúr-Kónya (2016), Baksa-Kónya (2019) and Baksa-Kónya (2021). Results here indicate how household heterogeneity modifies the previous picture, and what additional effects we find from the fact that the crisis hit wealthy and poorer households differently.

The main assumptions are as follows. The source of household heterogeneity is the presence of exogenous, idiosyncratic income shocks. Since households react to these shock with their consumption-saving decision, exogenous (labor) income heterogeneity translates into endogenous wealth differences. As mentioned above, possible wealth levels are bounded from below, which means that the ability of households to borrow - and hence adjust to a negative shock - is imperfect.

The solution algorithm is based on the assumption that there are no recurring aggregate shocks in the economy. Therefore, the long-run position of the economy can be described by a stationary equilibrium, where although the position of individual households changes each period, the aggregate wealth distribution is ergodic, and hence macroeconomic aggregates are constant.[2] It is possible, however, to study the effects of unforeseen, one-time shocks. In this case

[2] A classic reference that allows for aggregate shocks in this framework is Krussel-Smith (1998).

we first describe the stationary equilibrium before the shock, then calculate the new stationary equilibrium after the shock. It is also possible to study the transition between the initial and new steady states. This paper uses this approach to study the impact of the global financial crisis.

I model the financial crisis as a one-time, permanent tightening of financial conditions. This is achieved by increasing the borrowing constraint, where the extent of the increase is calibrated to the rise in the fraction of non-performing loans among Hungarian households after the crisis. After the shock I calculate the new stationary equilibrium, and compare it to the initial wealth distribution. The main conclusion is that due to the stronger precautionary behavior the aggregate capital stock rises, and the equilibrium interest rate falls. This is in line with the observed behavior of macroeconomic aggregates in Hungary, but the effects in the model are small. The (additional) impact caused by household heterogeneity is not insignificant, but quite modest compared with observed aggregate changes.

It is likely that short-run effects are larger, especially for vulnerable households. I use a simple method to quantify this. I assume that after the crisis shock the consumption-saving decisions of households can generally be well approximated by the decision rules under the new stationary equilibrium. There are, however, households whose wealth is below the new minimum level. For these agents I assume a faster "wealth convergence". Overall, I study howe consumption changes in the first period after the shock for different households and in the aggregate. Results indicate that for households whose wealth is too low, consumption drops on impact by 40%. For other households consumption does not decrease. Since the number of vulnerable households was initially fairly high, aggregate consumption falls by 3%, which is solely due to income heterogeneity.

The paper is structured as follows. Section 2 describes the model. Section 3 details the solution algorithm. The fourth section presents results from the crisis simulation: first it describes the initial, pre-crisis stationary equilibrium, then the new stationary equilibrium is presented, and finally the short-run consumption impact is quantified. Section 5 summarizes the main results, and discussed future research directions.

2 The model

The model uses the well-known Aiyagari (1994) approach, calibrated using Hungarian data. There is a large number of heterogenous households consuming and saving. The source of heterogeneity is the presence of idiosyncratic income shocks hitting each household. Although households are identical ex ante, different shock histories mean that incomes and wealth will differ ex post. Besides the exogenous shock, the other main determinant of income heterogeneity is the savings behavior of households, with which they react to the random events hitting them.

The other main assumption of the modeling framework is that borrowing opportunities are limited. I assume that while households can save any amount, they cannot become net debtors. Since there is only a single financial asset available for borrowing and saving, this means that this asset cannot be negative. In a framework with more assets, the analogous assumption would be that borrowing requires a collateral (for example real estate).

Household savings represent the supply side of the capital stock. Demand for capital comes from perfectly competitive firms, whose behavior can be described by a representative firm. In the stationary equilibrium the sum of individual savings have to equal capital demand .

2.1 Households

The problem of a typical household can be written as

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

$$\begin{aligned} \text{s.t.} \quad & c_t + x_t = z_t w_t + \left(r_t + \delta\right) a_t \\ & a_{t+1} = \left(1 - \delta\right) a_t + x_t \\ & a_{t+1} \geq 0, \end{aligned}$$

where c is consumption, w is labor income, a is net assets (capital), r is the net interest rate, x is savings (capital investment), and z is an idiosyncratic labor supply (or income) shock. The last inequality is the *borrowing constraint*, which implies that the net wealth of households (or their net capital stock) cannot be negative. When writing capital income we take into account depreciation, i.e. the value of the interest rate is net of capital amortization.

The labor supply shock is independent across households, but follows the same first-order Markov process:

$$\Pr\left[z_{t+1} = z_j | z_t = z_i\right] = p_{ij}.$$

Let P denote the constant transition probability matrix, where the typical element is p_{ij} and by definiton $\sum_j p_{ij} = 1$. Also, let Z denote the vector of labor supply outcomes at period t across households.

The exogenous evolution of the labor supply distribution is given by

$$Z_{t+1} = Z_t P.$$

I assume that the number of households is large enough that the cross-sectional distribution is given by the *ergodic distribution*, i.e. $Z_t = Z$. Let us further normalize the z shock provess so that $\mathbb{E}z = 1$.

The problem can be written in recursive form, using the following Bellman equation:

$$v(a, z) = \max_{a' \geq 0} \left\{ \log\left[zw + (1+r)a - a'\right] + \beta \mathbb{E}_{z'|z} v\left(a', z'\right) \right\},$$

where $v(a, z)$ is the value function of the problem. The first state variable is exogenous (the income shock), and the other is endogenous (household wealth). The solution to the problem is given by the *policy function*:

$$a' = g(a, z; r), \tag{1}$$

where a' denotes assets chosen for the next period. The optimal asset choice also depends on the aggregate state of the economy. We show below that the interest rate is a sufficient statistic for this purpose.

2.2 Firms

The problem of the representative firm is described by a simple, intratemporal profit maximization:

$$\max \pi_t = AK_t^{\alpha} N_t^{1-\alpha} - w_t N_t - (r_t + \delta) K_t,$$

where n is labor input, k is capital input, and r^k is the rental price of capital.

The first-order conditions are as follows:

$$w_t = (1-\alpha) AK_t^{\alpha} N_t^{-\alpha}$$
$$r_t + \delta = \alpha AK_t^{\alpha-1} N_t^{1-\alpha},$$

i.e. the marginal products of capital and labor equal their respective factor prices.

2.3 Equilibrium

To describe the equilibrium of the model, we need to study factor markets. Individual labor supplies $z_{j,t}$ are random variables. Since shocks are idiosyncratic, and the number of households is large enough, aggregate labor supply is known and constant. Let this be denoted by the variable N. Formally, we have that

$$N_t = N\mathbb{E}z = N,$$

since by our earlier assumption the expected value of the shock (which equals its cross-section average) is 1.

Using the labor market equilibrium, we can write down demand for capital as follows:

$$\frac{K_t}{N} = \left(\frac{\alpha A}{r_t + \delta}\right)^{\frac{1}{1-\alpha}}. \tag{2}$$

Note that on the left-hand side we write capital per person, which is seen to be a function of the equilibrium interest rate r_t. Using this equation and the firm first-order condition for labot we can also express the equilibrium wage rate as a function of the interest rate:

$$w_t = (1-\alpha) A \left(\frac{\alpha A}{r_t + \delta}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3}$$

To characterize capital market equilibrium, we need to aggregate the supply side across households:

$$K_t = \sum_a \sum_z a_t \mu_t (a_t, z_t), \tag{4}$$

where $\mu_t (a_t, z_t)$ denotes the *endogenous* distribution of household according to labor supply and wealth.

The solution of the model is the following competitive equilibrium:

1. Given factor prices (w, r), firms maximize profits.

2. Given factor prices, households decide optimally, given by the decision rule (1).

3. The labor market clears, i.e. labor demand equals the constant aggregate labor supply N, and the wage is given by equation (3)

4. Aggregate household assets (capital) equal capital demand, as described by equation (4).

5. The evolution of household distribution is given by equation

$$\mu_{t+1} (a_{t+1}, z_{t+1}) = \sum_a \sum_z \Pr [g (a_t, z_t), z_{t+1} | z_t, a_t] \mu (a_t, z_t),$$

   where we take into account that changes in household wealth are described by the optimal decision rule.

3 Solution method

The model is solved numerically. The main complication comes from household heterogeneity, intermediated via the savings behavior. Aggregating individual assets gives us the economy-wide capital stock, which in turn determines the equilibrium interest rate. Households, on the other hand, need to forecast the interest rate for their optimal savings choice, which includes forecasting the endogenous wealth distribution. This, unfortunately, means that the full problem is mathematically intractable.

3.1 Numerical algorithm

The algorithm targets finding the long-run, stationary equilibrium instead of the time-varying short run outcome. Since there is no aggregate uncertainty in the economy - labor supply shocks are idiosyncratic, uncorrelated and the number of households is large -, the economy will converge to the stationary equilibrium from any initial wealth distribution. The stationary equilibrium is described by constant aggregate variables, and the ergodic distribution of households:

$$r_t = r$$

$$K_t = K$$

$$w_t = w$$

$$\mu_t(a_t, z_t) = \mu(a, z).$$

Stationary equilibrium does not mean that the economy is static. Household income and wealth changes each period. This, however, only means changing position within the ergodic distribution: according to the law of large numbers, aggregate savings and the capital stock remain constant.

We can write down the solution algorithm as follows:

1. First, we solve the household problem for a given interest rate. This leads to the decision rule describing optimal savings behavior, $g(a, z; r)$.
2. Second, we simulate household behavior for a sufficiently large number of periods, given the policy function and the exogenous Markov process for labor supply shocks.
3. Over the course of the simulations the economy eventually reaches the stationery distribution for the given interest rate. Leaving out a suitable number of initial periods, we take the time-series average of aggregate capital supply, which are computed as the sum of individual assets in each period.
4. Steps 1-3 trace out aggregate capital supply as a function of the interest rate. Aggregate capital demand is given by equation (2). The equilibrium interest rate is the one where capital market is in equilibrium, i.e. aggregate capital demand equals aggregate capital supply.
5. With the equilibrium interest rate in hand, we can calculate the other economy-wide variables, such as the wage rate, the capital stock and the ergodic distribution of household income and wealth.

3.2 The method of endogenous gridpoints

I solve the household problem with the so-called *endogenous gird method* (EGM), introduced by Carroll (2006).[3] The method is based on the functional Euler equation derived from the Bellman equation of the household problem. This can be written as follows:

$$\frac{1}{c} = \beta \left(1+r\right) \sum_{j=1}^{n_z} \frac{\pi_{ij}}{c'} \tag{5}$$

$$c = wz + \left(1+r\right) a - a'. \tag{6}$$

Moreover, we can write the policy function for the optimal consumption choice as

$$c = wz + \left(1+r\right) a - g\left(a,z\right) = h\left(a,z\right).$$

First, we discretize the state space for assets, and - according to the Markov assumption - the state space for labor supply. Let these are given by the sets $A = [0, \ldots, a_{max}]$ and $Z = [z_{min}, \ldots, z_{max}]$

Next, we select an initial policy function for consumption, $h_0\left(a,z\right)$. Substituting the policy rule into the right-hand side of equation (5) for $c' = h_0\left(a', z'\right)$, we can calculate current consumption c for a given $z_i \in Z$ and $a'_l \in A$ wealth level:

$$\frac{1}{c_{l,i}} = \beta \left(1+r\right) \sum_{j=1}^{n_z} \frac{\pi_{ij}}{h_0\left(a'_l, z'_j\right)}.$$

Using this in equation (6), we can calculate the implied starting level of assets:

$$a_{l,i} = \frac{c_{l,i} + a'_l - wz_i}{1+r}.$$

The $\{a_{l,i}\}$ initial asset levels are the *endogenous grid points* in which we determine the updated decision rules for savings and consumption. Since $\{a_{l,i}\}$ are not necessarily elements of the set A, we need to interpolate. For simplicity and computation speed I use linear interpolation. Let us denote the continuous interpolation function for assets with $\tilde{a}_i\left(a_k\right)$, where $a_k \in A$.

The last step is to recalculate savings and initial consumption in the original, exogenous grid

[3] A similar, alternative solution method is value function iteration. The main advantage of the EGM algorithm is that it is significantly faster.

points:

$$\begin{aligned} a'_{k,i} &= \max\left\{\tilde{a}_i\left(a_k\right), 0\right\} \\ c_{k,i} &= (1+r)\,a_k + wz_i - a'_{k,i}. \end{aligned} \tag{7}$$

Note that in the savings rule we essentially invert the relationship between (a, a'), taking into account the borrowing constraint, i.e. that assets cannot be negative.

Equation (7) defines a new, updated policy rule $h_1(a, z)$ for consumption, which we can compare to the original, $h_0(a, z)$ decision rule. If the suitably defined distance between the two is smaller than a pre-determined tolerance level, we accept the policy rule h_1 as the solution to the household's problem. When the two decision rules are different, we update our guess to h_1 and restart the process. We iterate until the update to the decision rule is smaller than the tolerance level.

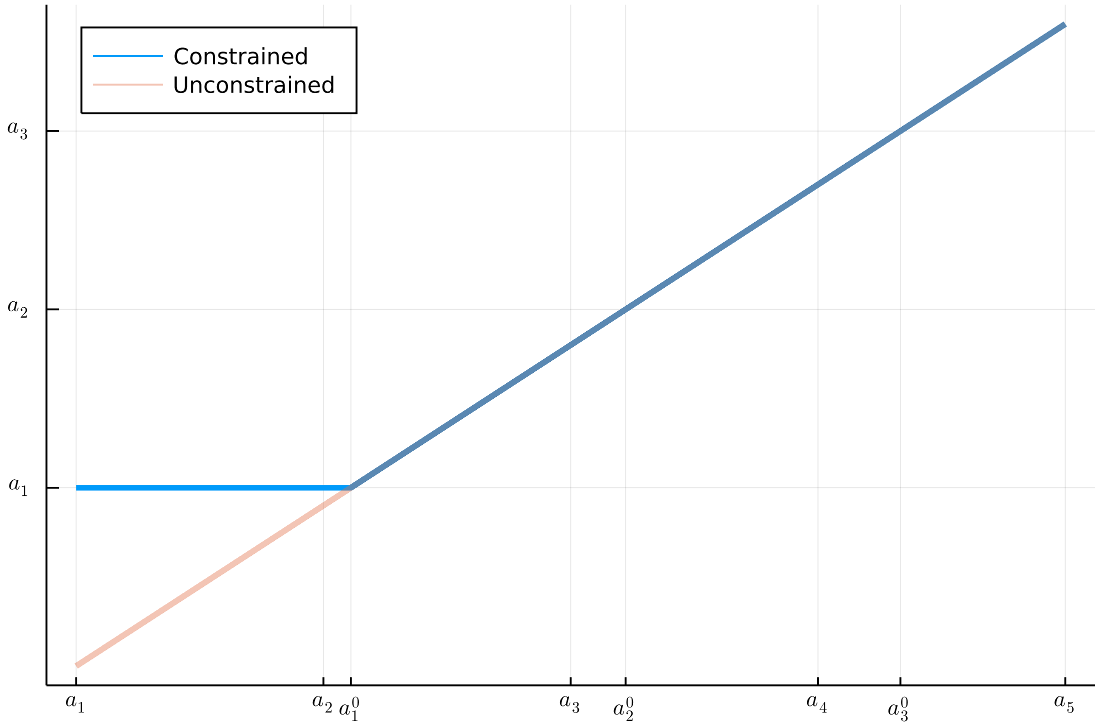
The relationship $h_0 \to h_1$ is called the Coleman operator (Coleman, 1990), and can be written as

$$h_t(a, z) = \Omega \circ h_{t-1}(a, z)$$

for an arbitrary iteration step t. The optimal decision rule of households is thus a fixed point of the Ω operator. The operator is function values, hence we need to find the fixed point - in the steps describe above - with a discrete approximation. It can be shown that the fixed point is unique, and the iterative process converges to this unique function from any initial policy rule.

The logic behind the endogenous grid method is illustrated on Figure 1. The exogenous, initial grid points - where the policy rule is defined - are denoted by $\{a_1, a_2, \ldots\}$. Using the Coleman operator, we invert the policy rule, and for each grid point on the vertical axis, we find the corresponding initial, endogenous wealth levels. These are shown on the horizontal axis as points $\left\{a_1^0, a_2^0, \ldots\right\}$. We need the updated policy rule, however, in the original grid points. Therefore, we interpolate the decision rule defined in the $\left\{a_l^0\right\}$ points for the grid points $a_k \in A$. In this step, we need to take into account the borrowing constraint: on the figure, initial asset levels a_1 and a_2 (on the horizontal axis) would imply negative assets for next period (on the vertical axis). We thus need to exchange this part of the policy rule with $a' = 0$. The final, updated decision rule is shown in blue on the figure, now incorporating the borrowing constraint

Fig. 1: The endogenous grid method



as well.

3.3 Calibration

For the numerical solution of the model, we need to calibrate various parameters. We set $\beta = 0.95$, which is a standard value in the literature at an annual frequency. The depreciation rate is chosen to be $\delta = 0.08$, also often used in the literature. The elasticity of the production function with respect to capital is set to $\alpha = 0.4$, which corresponds to the time series average of the annual Hungarian wage share figures (Kónya, Krekó and Oblath, 2020).

The main challenge for the calibration is to chose the labor supply grid points, and to quantify the corresponding probability transition matrix. Note that given the equilibrium wage rate, the individual labor supply shocks also indicate labor income. We thus use data on labor income from Eurostat, where these are reported by income deciles. The data contains both average labor income for a given decile, and transition probabilities among different deciles. The exact data sources are the following:

1. "Transitions of income within one year by decile" - Eurostat Table `ilc_di30a`.

2. "Distributions of income by quantiles – EU-SILC and ECHP surveys" – Eurostat Table `ilc_di01`.

The exogenous labor supply distribution is chosen to replicate the income distribution in 2007. The values are relative to the full sample average, i.e. we work with relative income levels. Based on the 2007 observations, the distribution of (relative) labor supply is as follows:

$$z = [0.35, 0.51, 0.64, 074, 0.84, 0.94, 1.04, 1.18, 1.41, 2.37] .$$

To calibrate the transition probabilities, we take values from 2006-2007. Unfortunately, Eurostat does not report the full matrix, but only five transition categories:

1. a fall of more than one decile,
2. a fall of one decile,
3. no change,
4. a rise of one decile,
5. a rise of more than one decile.

For the two extreme cases (1 and 5), I assume that the change means either 2 or 3 deciles. I split the category into 2 and 3 decile changes following an ad-hoc 2/3-1/3 rule, based on the assumption that smaller income changes are more likely. In cases where a change of more than 2 deciles are not possible (such as a more than 2 decile fall for someone initially in the third decile), I naturally assigned all the change to a 2 decile change. These considerations lead to the following transition matrix:

$$P = \begin{bmatrix} 0.49 & 0.24 & 0.18 & 0.09 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.21 & 0.31 & 0.19 & 0.16 & 0.13 & 0 & 0 & 0 & 0 & 0 \\ 0.11 & 0.18 & 0.29 & 0.15 & 0.14 & 0.13 & 0 & 0 & 0 & 0 \\ 0.04 & 0.09 & 0.19 & 0.26 & 0.17 & 0.14 & 0.11 & 0 & 0 & 0 \\ 0 & 0.07 & 0.12 & 0.19 & 0.24 & 0.17 & 0.13 & 0.08 & 0 & 0 \\ 0 & 0 & 0.07 & 0.12 & 0.14 & 0.23 & 0.23 & 0.13 & 0.08 & 0 \\ 0 & 0 & 0 & 0.1 & 0.16 & 0.16 & 0.25 & 0.2 & 0.08 & 0.05 \\ 0 & 0 & 0 & 0 & 0.08 & 0.14 & 0.14 & 0.3 & 0.26 & 0.08 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0.16 & 0.16 & 0.33 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.12 & 0.16 & 0.16 & 0.56 \end{bmatrix}$$

For the numerical solution algorithm, we also need to select the grid points for the endogenous state variable. I choose 1000 grid points on the interval $[a_{min} \ldots a_{max}]$, where a_{min} is given by the borrowing constraint, and $a_{max} = 40$ is a large value (about 7 times the per capita steady state in the representative agent benchmark, see below).[4] Since solution accuracy is more sensitive to low values (where the borrowing constraint binds), I assign grid points more densely in this region.

As explained earlier, capital supply is calculated by simulating individual decisions and aggregating across households and over time. I set the number of households to $N = 1000$, and simulate individual decisions for $T = 300$ periods. Over this horizon the cross-sectional distribution of assets reaches the ergodic state. I calculate the time series average - after aggregating across households - for periods $t = 201 \ldots 300$. This gives us capital supply for a given interest rate, taken as given by individual households. I set the initial level of assets to a common value across all households, which equals average wealth in the representative agent benchmark. Since the stationary equilibrium and the ergodic distribution of households are unique, the choice of an initial wealth distribution only affects the speed of convergence.

The simulations and the solution algorithm are implemented in Julia.[5] The advantage of using Julia compared to other software packages is twofold. First, Julia is free and there are many

[4] Results are robust to reasonable variations in the upper bound.
[5] https://julialang.org/

supplementary packages available for macroeconomic research. One such package is QuantEcon,[6], which also includes teaching material for an advanced macroeconomic course. The second advantage of Julia is speed: with appropriate programing, Julia is almost as fast as lower-level languages used for such purposes in the past, such as the Fortran implementation in Aiyagari (1995). Using Julia, on the other hand, is much simpler. Its syntax is very similar to high-level matrix languages, such as Matlab or Octave.

4 Results

After presenting the model and the solution algorithm, I now turn to simulating the income ad wealth distribution. First I study the period before the financial crisis of 2008-2009, which gives the model baseline. Next, I look at the long-run and short-run effects of the financial crisis.

4.1 Before the crisis

The calibration discussed in the previous section uses this period, particularly for the income distribution and the transition probabilities. Let us now see the implied long-run equilibrium for the calibrated economy.

4.1.1 Representative household

As a benchmark it is useful to calculate the equilibrium aggregates for a version of the model without household heterogeneity. The interest rate and the capital stock are given by the following equations:[7]

$$\bar{r}^{rep} = \frac{1}{\beta} - 1$$
$$\bar{K}^{rep} = \left(\frac{\alpha}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\alpha}} N.$$

The calibration above implies that $\bar{r}^{rep} = 0.0526$ and $\bar{K}^{rep}/N = 6.3$.

To determine labor supply, we need to calculate the ergodic distribution of the Markov pro-

[6] https://julia.quantecon.org/

[7] Both here and in the next sections equilibrium quantities are presented in a per capita form. Values for the total economy can simply be calculated by multiplying the per person values with the number of households, chosen to be $N = 1000$ in the simulations.

cess, which is

$$p_z^{erg} = \begin{bmatrix} 0.06 & 0.07 & 0.09 & 0.10 & 0.12 & 0.12 & 0.13 & 0.12 & 0.10 & 0.09 \end{bmatrix}. \tag{8}$$

Using this, the implied labor supply in the heterogenous agent model is $N = 1.03$.[8] Assuming the same labor supply for the representative agent version, total capital stock is $\bar{K}^{rep} = 6.49$. Finally, total output - given by the production function - is $\bar{Y}^{rep} = 2.15$, which implies a long-run capital-output ratio of $\bar{K}^{rep}/\bar{Y}^{rep} = 3.02$.

The ergodic distribution of labor supply - and hence labor income - is somewhat different from the calibration assumption, where information about income deciles was used. This is not surprising, since the Hungarian economy was buffeted by large shocks over a brief time period even before the financial crisis, and hence initial stationarity is a strong assumption. That said, the ergodic distribution is fairly close to a uniform one, which is the definition of deciles. Relative to the uniform distribution, we find somewhat fewer households at low income levels, and somewhat more households at medium income levels.

4.1.2 Heterogenous households

Let us now turn to predictions from the heterogenous household case. Here we only have numerical results, based on the solution algorithm discussed above. The equilibrium interest rate is $\bar{r}^{het} = 0.0463$, which is lower than for the representative agent benchmark. The equilibrium capital stock is $\bar{K}^{het} = 7.03$, which is higher than in the model without heterogeneity. Finally, output and capital-output values are given by $\bar{Y}^{het} = 2.22$ and $\bar{K}^{het}/\bar{Y}^{het} = 3.17$.

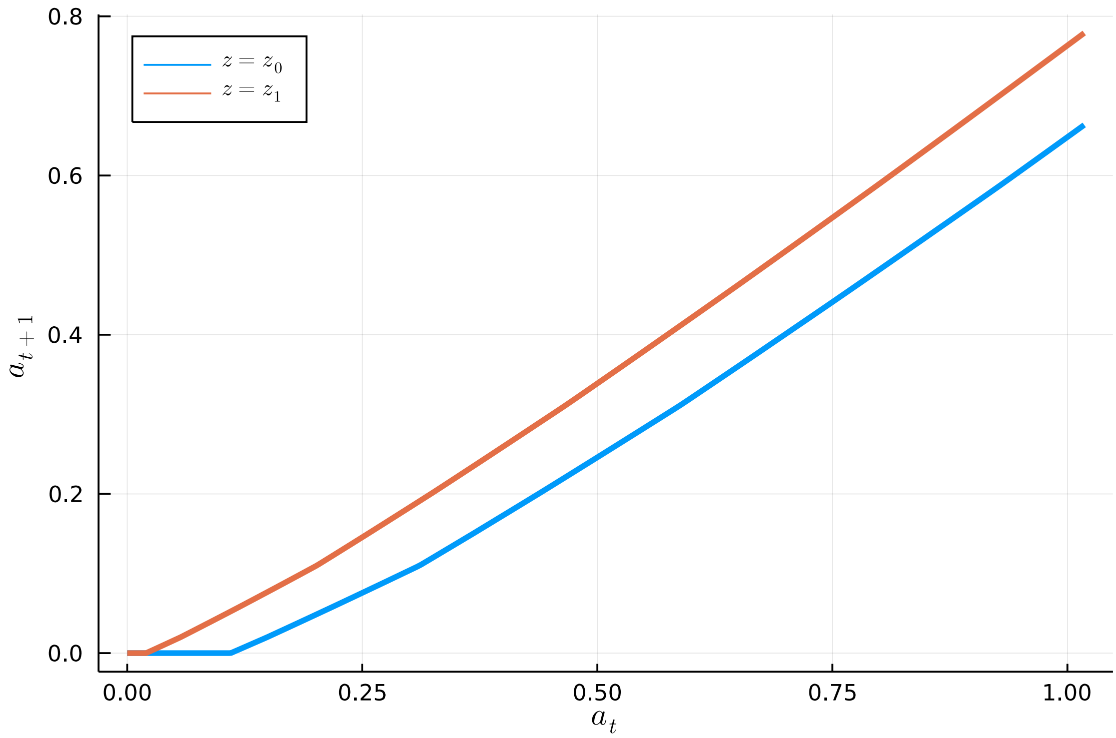
What is behind these differences? In the representative household case, there is no borrowing and lending, since all households are the same and they all make the same decisions. This means that assets (capital) are the same for each household. In the heterogenous case, however, assets also serve as insurance against negative idiosyncratic shocks, which lead to a decline in income. Precautionary savings behavior would not lead to sizable changes without the presence of a borrowing constraint. With assets bounded from below, households want to insure themselves against a series of negative income shocks, which would take them close to the borrowing

[8] In the calibration labor supply was determined by incomes for particular deciles relative to the population average. If the ergodic distribution of labor supply was uniform, as assumed in the calibration, total labor supply would be 1. The difference is from the actual value is small, because the ergodic distribution is not very far from being uniform.

constraint, hitting which would prevent them from further consumption smoothing. Overall, idiosyncratic shocks and the borrowing constraint lead to the result that average wealth - and hence average capital - are higher, than for the representative household. From this it directly follows that the equilibrium interest rate is lower, since it id determined by the marginal product of capital.

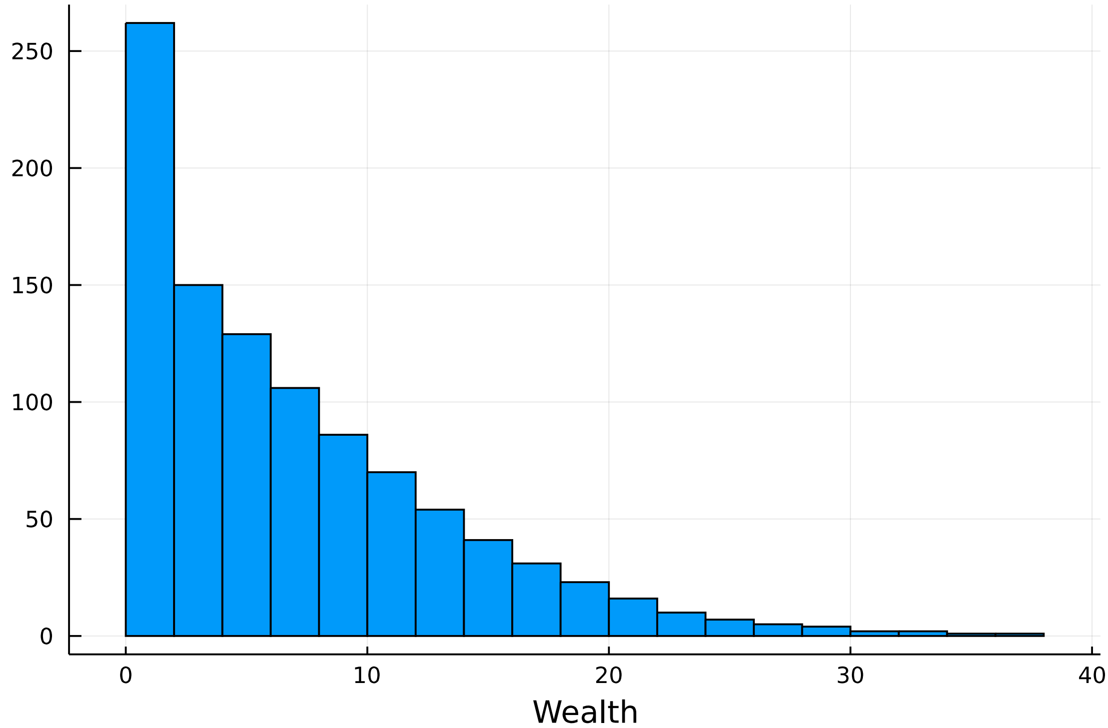
Note that the differences are quite sizable. The reference study (Aiyagari, 1995) finds much smaller deviations from the representative agents case (Table II). There are two reasons why in the current case we find bigger differences, both caused by the properties of the labor supply shock. In the current calibration the shock is fairly persistent, and income fluctuations are sizable. This means that households need a bigger buffer against income risk. It is more likely that income drops significantly, and if a household finds itself with low income, it has a bigger chance to remain in that position. According to Aiyagari (1995), both effects imply that the equilibrium interest rate falls (relative to the representative agent benchmark).

Fig. 2: Policy functions for wealth under low income



Let us see the conditions under which the borrowing constraint binds. Figure 2 plots decision rules for next period assets when the labor supply shock is the lowest (z_0) or the second lowest (z_1). The figure shows that the borrowing constraint only binds for very low current asset levels, mostly when labor supply is the lowest. This may seem surprising, but it follows directly from household precautionary behavior. Since households want to avoid a binding borrowing constraint, they hold enough wealth to help them over even an extended period of low income. For the borrowing constraint to bind when labor supply is the lowest, assets have to be below $\underline{a} = 0.096$, which is very low compared to average assets ($\bar{a}^{het} = 7.03$). It is an important question, of course, to see what fraction of households have such a low level of assets.

Fig. 3: Household wealth distribution

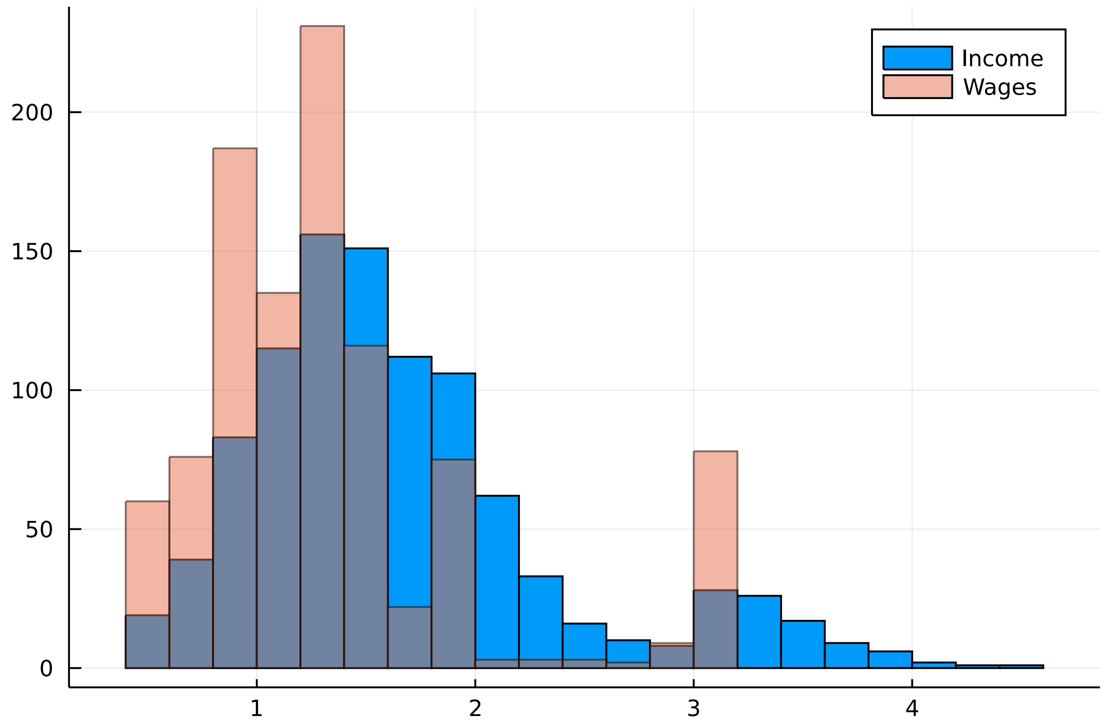


The ergodic distribution of household wealth is presented on Figure 3. The distribution is quite skewed: median wealth equals 5.76. Only a few households hold a lot of assets, but the majority of households have fairly significant wealth holdings (relative to the condition of being borrowing constrained). In any given period, out of the 1000 households only 15 is subject to the borrowing constraint, based on the ergodic distribution. This is only 1.5% of all the households. Moreover, this state is not very persistent, and constrained households escape in a few periods.

Besides the distribution of wealth, it is instructive to also look at the distribution of labor income and total income, where the latter also includes returns to asset holdings. These are illustrated on Figure 4, with the two distributions overlaid on each other. For labor income the histogram simply shows the values in z, together with the p_z^{erg} ergodic probabilities (equation

[8]).

Fig. 4: The distribution of labor income and total income



Total income is the sum of labor and capital income:

$$y_{j,t} = \bar{w} z_{j,t} + \bar{r} a_{j,t}.$$

Factor prices do not change over time, since we are looking at the stationary equilibrium. In the distribution of total income, wealthy households have a higher weight, since they have significant capital income. Note, however, that for most households capital income is not very important. Average labor income is 1.33, and 99% of households have a capital income below this level. This, of course, if in line with reality, since only a very small fraction of households hold enough assets that yields significant capital income.

4.2 The effects of the financial crisis

I assume that the main transmission mechanism of the financial crisis was a tightening of the borrowing constraint. This is implemented the following way. The Stability Report of the National Bank of Hungary (MNB, 2012) states that the share of non-performing household loans increased from 1.5% to 16% between 2008 Q2 and 2012 Q2. I calibrate the increase in the borrowing constraint to this statistic, i.e. I select the wealth level below which 16% of households fell before the crisis (according to the ergodic distribution). This value is $\underline{a}^{post} = 0.83$. The borrowing constraint therefore changes to

$$a_{t+1} \geq \underline{a}^{post}$$

after the crisis. All other parameters remain the same as before.

4.3 The new stationary equilibrium

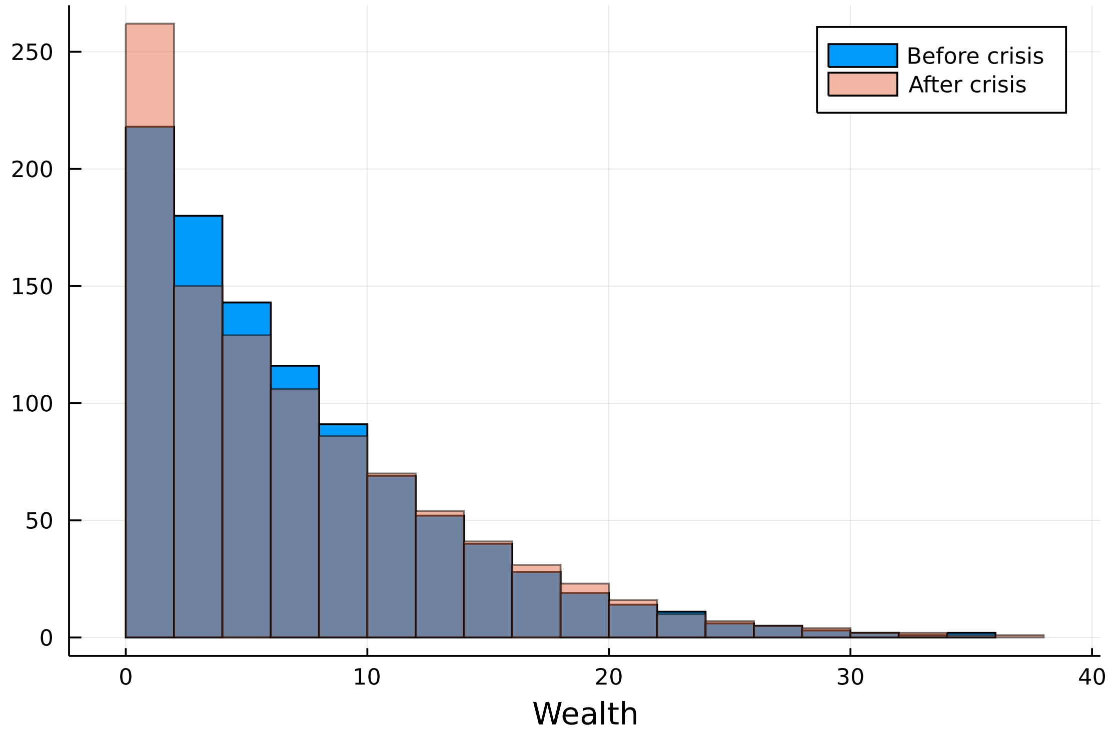
The new equilibrium interest rate is somewhat lower, $\bar{r}^{post} = 0.0458$. The equilibrium capital stock rises to $\bar{K}^{post} = 7.08$. With the new, higher borrowing constraint the precautionary saving motive becomes stronger, since households have to hold more wealth to avoid hitting the borrowing limit. The effects are small: the capital stock rises by 0.7%. GDP increases by even less, since it is proportional with the α power of the capital stock. Tighter financial conditions thus lead to an "investment boom" in the long run, but the quantitative extent of this move is negligible.

The new wealth distribution is shown on Figure 5, compared to the pre-crisis distribution. Although not visible on the chart, recall that the minimal wealth level post-crisis is $\underline{a}^{post} = 0.83$. This explains why the number of households with very little wealth decreases, but the share of households with medium wealth rises. Interestingly, the share of high net worth households does not increase, probably because for these households the probability of running into the borrowing constraint remains negligible.

Boldizsár et al. (2016) studied the wealth distribution of Hungarian households in detail, using the 2014 Household Finance and Consumption Survey (HFCS). They find a Gini coefficient of 0.58. According to the simulations, the Gini was 0.49 before the crisis, and falls to 0.45 after the crisis. The reason behind the decline is that low net worth households increase their asset holdings by proportionately more in order to avoid the higher borrowing constraint.

It is well known that models of this type have trouble to fully replicate the empirical concentration of wealth. That said, there were additional shocks hitting the Hungarian economy, whose effects countered the crisis impact studied in this paper. First, the economy was hit in

Fig. 5: Wealth distribution under the higher borrowing constraint

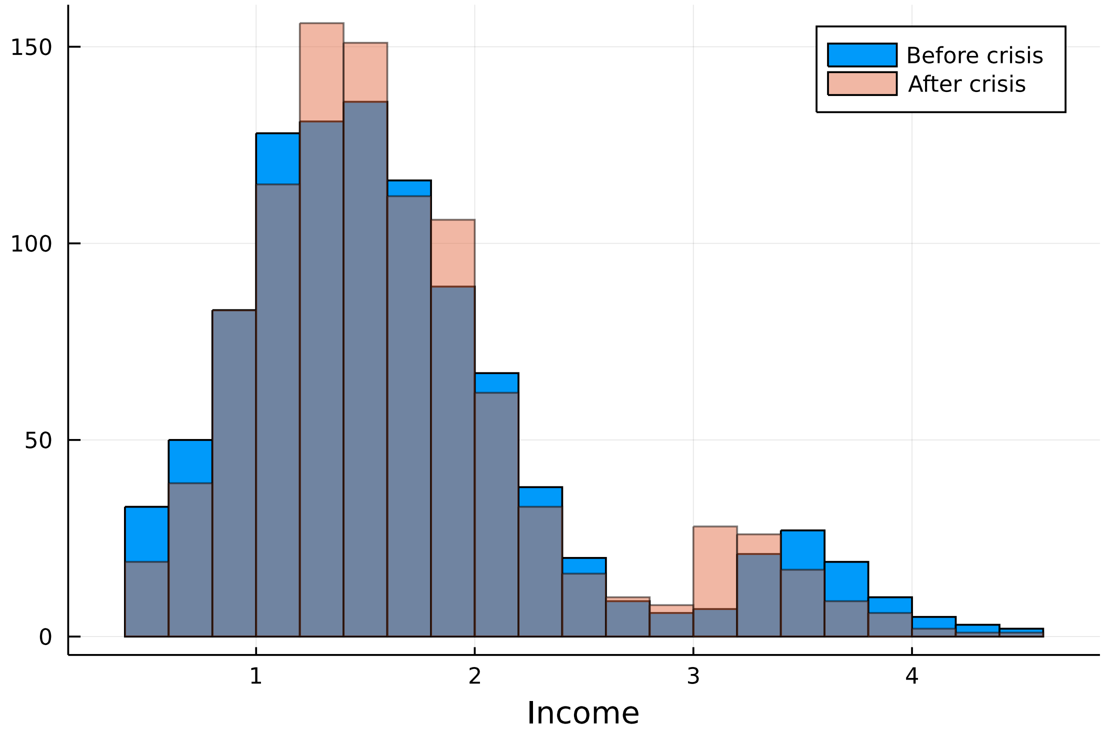


2012 by the second wave of the financial crisis in Europe. Second, personal income tax rates fell significantly, especially for high income households. This latter shock can be studied in the current model framework, and can be a subject for future research.

Let us know see what happened to the number of constrained households, and with the condition to fall in this category. Using the policy rule for assets, the constraint is effective when (i) labor supply takes on its lowest value, and the level of wealth is below 0.94, or (ii) labor supply takes on its second lowest value and the level of wealth is below 0.85. Using the new ergodic distribution, 51 out of 1000 households have assets below 1, which is 5.1% of the population. This is still a low value, but higher than the previous 1.5% - a significant increase of about 300%.

Figure 6 shows the distribution of income before and after the crisis. Note that for labor income the two distributions are the same, since the exogenous stochastic shock does not change. There are no major differences for total income either. The number of very low income households is somewhat lower, since their wealth is now higher due to the increased borrowing constraint.

Fig. 6: The distribution of income under the higher borrowing constraint



4.4 The impact of the crisis on consumption

The new stationary distribution shows how the economy adjusts to the crisis shock over time. It is also worth studying, however, how households adjust in the short run. A full investigation would require the computation of the transition path between the old and new stationary equilibria (Kirkby, 2017). Here we chose a simpler approach, which can give us a quick answer to how the consumption of heterogenous households changes, with a particular attention to those with low wealth.

I use the following procedure. The tightening of the borrowing constraint directly impacts households whose wealth is suddenly below the new limit, $a_i < \underline{a}^{post}$. Clearly all other households are also impacted indirectly, first because of changes in aggregate variables, and also because they might be subject to the new constraint in the future. For these latter households, I simply assume that their behavior is given by the new, stationary decision rules. Since this takes into account the new borrowing constraint, these households will not hold less wealth in the future.

For households with initially lower assets I assume that they will try to reach the new minimum wealth level as fast as possible. Let a_0 and c_0 denote assets and the consumption choice when the crisis hits, while let a_1 denote the new, chosen level of assets. Furthermore, let $\Delta_0 = \bar{a}^{post} - a_0$ indicate the shortfall in assets relative to the new borrowing constraint. Finally, let $h^{post}(a, z)$ stand for the new stationary decision rule, extended to include wealth levels that are no longer feasible. Using this notation, I postulate the following consumption rule for households with two little initial wealth:

$$c_0^{low} = \max\left\{h^{post}\left(\underline{\mathrm{a}}^{post}, z_0\right) - \Delta_0, \lambda h^{post}\left(\underline{\mathrm{a}}^{post}, z_0\right)\right\}.$$

The interpretation of the equation is the following. When the income and wealth of the households are sufficient, consumption falls on impact so that the household can immediately save the "missing wealth". If this leads to negative consumption, I allow the household to increase its assets more slowly. In particular, I assume that consumption must at least reach a subsistence level. This is controlled by the parameter λ, which - in the absence of better information - I set to 0.66, i.e. consumption of such households falls by $1/3$ relative to households that are also poor, but whose assets are above the new borrowing constraint.

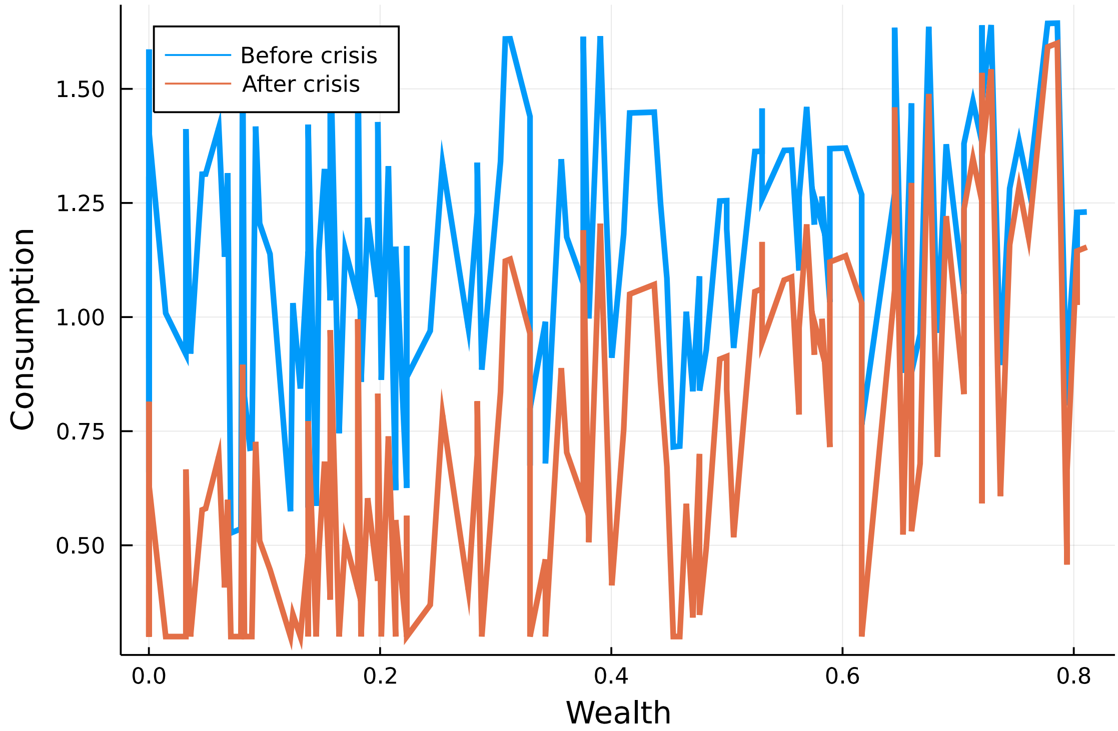
The calculations are for one period only, which measures the immediate, very short run effect of the crisis. It would of course be possible to simulate more periods, and see how the balance sheet adjustment of the most impacted households takes place.

The main results can be summarized as follows. Total, aggregate consumption falls by 3.2%, which is a fairly significant effect. Within this, the fall of consumption is almost 40% among households whose initial wealth is below the new borrowing limit. This is a function of the parameter λ, but the fall is to a larger extent due to households who have enough income to fill the wealth gap fully. It is interesting to see that consumption for those households who are not directly impacted does not change on average.

Figure 7 plots the consumption choice of low wealth households, compared to the decision rule of the same households without the crisis. The significant differences in consumption levels are due to the fact that the labor income wz of these households can be quite different. Although wealth levels and actual income are positively correlated, the correlation coefficient is not too high (around 0.35). This means that some low wealth households do not have very low incomes.

The figure shows that among all impacted households, only a small fraction is unable to cover the required increase in wealth (the horizontal section of the red line). Most households have

Fig. 7: The direct impact of the crisis on households with low wealth



enough income to fill the gap, but many experience large drops in consumption as a result.

It is important to acknowledge the main limitations of the exercise, which is the usage of the new stationary decision rules. This means that factor prices are perceived to be at the new, long run equilibrium levels. This may lead to errors in the calculation, especially when convergence to the new ergodic state is slow. It is beyond the scope of the current paper to analyze this, and I relegate the problem to future research.

5 Summary

This paper built a heterogenous household macroeconomic model to study how the global financial crisis impacted the wealth distribution and the short run consumption decisions of Hungarian households. The modeling framework is a well-known approach, which assumes idiosyncratic income shocks and incomplete financial markets (a borrowing constraint). The model was calibrated to Hungarian data from 2007, especially for income distribution and the transition probabilities across different income states.

Results from model simulations are as follows. The Hungarian wealth distribution is fairly skewed, with the majority of households holding relatively little wealth. That said, the logic of the model implies that only a small fraction of households is subject to the borrowing constraint. The reason is that households try to avoid this possibility, and build up a sufficient wealth buffer against realizations of negative income shocks.

To study the impact of the financial crisis, I assumed that there is a permanent tightening in the borrowing constraint. This leads to a necessary balance sheet adjustment for many households. The new, long run wealth distribution is not very different from the old one. The number of households with very little wealth falls somewhat, which is a direct consequence of the tighter borrowing constraint. Interestingly, the number of high net worth households also falls a bit.

Finally the paper also looked at the short run impact of the financial crisis on consumption. Results showed that low wealth household directly impacted by the tighter budget constraint cut back their consumption levels significantly. The aggregate consequence of this is a 3% drop in total consumption. Therefore, taking into account household heterogeneity is important from an aggregate perspective, especially when looking at short-run effects.

The most important open question is treatment of short-run transition in a more model consistent way. This requires the implementation of an algorithm, which takes into account the gradual change in factor prices towards the new stationary equilibrium. Such an algorithm is available in the literature, and its incorporation into the paper is the subject of future research.

References

Aiyagari, R. (1994). „Uninsured idiosyncratic risk and aggregate saving." *The Quarterly Journal of Economics*, **109**: 659-684.

Baksa D. és Kónya I. (2021). „Convergence stories of post-socialist Central-Eastern European countries." Manchester School, Forthcoming.

Baksa D. és Kónya I. (2019). „Convergence, productivity and debt: the case of Hungary." KTI Discussion Papers, MT-DP - 2019/16.

Benczúr P. és Kónya I. (2016). „Interest Premium, Sudden Stop, and Adjustment in a Small Open Economy." *Eastern European Economics*, **54**: 271-295.

Boldizsár A., Kékesi Zs., Kóczián B. és Sisak B. (2016). „The Wealth Position of Hungarian Households based on HFCS." *Financial and Economic Review*, **15**: 115-150.

Carroll, C. (2006). “The method of endogenous gridpoints for solving dynamic stochastic optimization problems.” *Economics Letters*, **91**: 312–320.

de Ferra, S., K. Mitman and F. Romei (2020). „Household heterogeneity and the transmission of foreign shocks.” *Journal of International Economics*, **124**: 103303 (online).

Huggett, M. (1997). „The one-sector growth model with idiosyncratic shocks: Steady states and dynamics.” *Journal of Monetary Economics*, **39**: 385-403.

Kaplan, G., B. Moll és G. L. Violante (2018). „Monetary Policy According to HANK.” *American Economic Review*, **108**: 697-743.

Kaplan, G. és G. L. Violante (2018). „Microeconomic Heterogeneity and Macroeconomic Shocks.” *Journal of Economic Perspectives*, **32**: 167-194.

Kirkby, R. (2017). „Transition paths for Bewley-Huggett-Aiyagari models: Comparison of some solution algorithms.” Working Paper Series 5642, Victoria University of Wellington, School of Economics and Finance.

Kónya I., Krekó J. és Oblath G. (2020). „Labor shares in the old and new EU member states - Sectoral effects and the role of relative prices.” *Economic Modelling*, **90**: 254-272.

Krusell, P. és A. A. Smith (1998). „Income and Wealth Heterogeneity in the Macroeconomy.” *Journal of Political Economy*, **106**: 867-896.

Marcet, A., F. Obiols-Homs és P. Weil (2007). „Incomplete Markets, Labor Supply and Capital Accumulation.” *Journal of Monetary Economics*, **54**: 2621-2635.

MNB (2012). *Jelentés a pénzügyi stabilitásról.* Magyar Nemzeti Bank, 2012 November.

Öztunali, O. és O. Torul (2018). „On income and wealth inequality in Turkey.” *Central Bank Review*, **18**: 95-106.

Tóth I. Gy. (2013).”Jövedelemeloszlás 2009-2012 között”. In: *Egyenlőtlenség és polarizálódás a magyar társadalomban.* Szerk: Szívós P. és Tóth I. Gy. TÁRKI Monitor Jelentések 2012.