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Introducing flexible retirement : a dynamic model

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ABSTRACT

Typically economists arguing for flexible (or variable) retirement age, but they rely on steady state analysis. In this paper we consider the replacement of a mandatory retirement system with a flexible one in real time. We show that even if early retirement is duly punished, diminishing the effective retirement age by 1 year raises the first year's and the total expenditures during transition by 8% and 70% of the original annual expenditure, respectively.

JEL codes: H11, H55

Keywords: retirement age, flexible retirement age, variable retirement age, transition cost

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A rugalmas korhatár bevezetése : egy dinamikus modell

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ÖSSZEFOGLALÓ

A közgazdászok általában támogatják a rugalmas (változó) nyugdíjkorhatárt, de állandósult állapotot elemeznek. Ebben a dolgozatban a kötelező korhatárú nyugdíjrendszer helyett egy rugalmast vezetünk be, valós időben. Megmutatjuk, hogy még ha az előrehozott nyugdíjba vonulást kellően büntetjük, a tényleges nyugdíjkor 1 éves csökkentése az első évi, illetve teljes költsége rendre az eredeti éves kiadás 8, illetve 70%-a.

JEL: H11, H55

Kulcsszavak: nyugdíjba vonulási kor, rugalmas nyugdíjkorhatár, változó nyugdíjkorhatár, áttérési költségek

Introducing flexible retirement: a dynamic model

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Abstract

Typically economists arguing for flexible (or variable) retirement age, but they rely on steady state analysis. In this paper we consider the replacement of a mandatory retirement system with a flexible one in real time. We show that even if early retirement is duly punished, diminishing the effective retirement age by 1 year raises the first year's and the total expenditures during transition by 8% and 70% of the original annual expenditure, respectively.

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1 Introduction

Since the 1990s, in the developed countries, there has been a tendency to raise the *full benefit* (normal, statutory etc.) retirement age (for short, FRA) in parallel with the rise of life expectancy. This measure in itself is futile if the *effective* retirement age does not increase similarly (from Gruber and Wise, eds. 1999 to Chłoń–Domińczak et al. 2021). Considering new EU countries, Gál and Radó (2020, Figure 16.5) demonstrated that in the foregoing countries, the two ages grew in parallel between 1996/2000 and 2014, easing the public burden.

According to economic theory, the best method to achieve such a parallel increase is to introduce an actuarially fair benefit–retirement age schedule: if a worker retires one year earlier/later than normal, then her annual benefit is to decreased/increased so that her lifetime net contribution remain 0. The simplest fair method is the so-called Nonfinancial Defined Contribution (NDC, e.g. Sweden, see Appendix 3 to this paper), but other methods work similarly well. These methods combine efficiency and fairness, though self-criticism has recently emerged concerning the neglect of the strong positive correlation between life expectancy and lifetime earnings (Breyer and Hupfeldt, 2009; Holzmann et al., eds., 2020).

In contrast, several new EU countries have been applying rigid methods concerning early retirement. In Czechia only those workers are allowed to retire before reaching FRA who have at least 35 years of contributions and even they are severely punished (OECD, 2020, pp. 25-29 and Appendix 3 to this paper). In Slovakia, there is a condition for early retirement: the initial benefit be at least as high as the minimum wage, approximately equaling to the average pension benefit of the year. The Polish retirement age policy is contrversial, for example, the effective retirement age frequently falls below the minimum age (Chłoń–Domińczak, 2019 and Appendix 3 to the paper). Each system applies too strict/loose requirements and diminishes social welfare with respect to a truly flexible system.

Hungary has been operating a dual, loose/rigid system since 2011. First, every female is allowed to retire without any benefit reduction if she acquired at least 40 years of entitlement (correlated but not identical to years of contributions). Second, nobody else is allowed to retire, even with reduced benefits, until she/he reaches the FRA, increasing otherwise quite fast: 62 (2013), 63 (2016), 64 (2019) and 65 (2022). Such a system is unfair to the excluded majority and socially suboptimal. Consider two females in 2021: one is 64 years old and has 39 years of entitlement, she has to wait another six months to retire. The other is only 60 years old but has 40 years of entitlement, therefore she can retire without any other limitation and draw full benefits for another 5 years, when she reaches her FRA (and after).

Why are such rigid/loose systems in force at all? Probably these governments do not trust market incentives to keep average retirement age close to the FRA and they enjoy making positive discrimination. But there are real problems with the flexible system as well. We name four problems: (i) the bulk of the workers do not know the rules, and eventually a large part may have opted for continued work if they knew the rules (Barr and Diamond, 2008, Section 4.7; Benitez-Silva et al., 2009). (ii) Even if workers knew the rules, they would retire at the earliest age (Barr, 2006). (iii) The healthier workers retire later and live longer, therefore they gain, while others lose (Fabel, 1994; Diamond, 2003; Eső and Simonovits, 2002; Simonovits, 2004; etc.). (iv) Replacing the rigid retirement age, but fixing the contribution rate, temporary deficits arise. These factors frequently strengthen each other: for example, due to recent real wage explosion and wage indexation of benefits in progress, in Hungary, early retirees lose with respect to workers delaying their retirement but they cannot help, due to (i) and (ii) (Simonovits, 2019).

In this paper we shall confine our attention to issue (iv). Using very strong homogeneity assumptions on demography and economy, we show that if all workers retire at 64 rather than 65, then the temporary and the total costs of this reform are equal to 8 and 70% of the pre-reform one-year expenditure, respectively. (In the simplest static model, the welfare gain is equal to 0.4%.) Relaxing our very strong assumptions the results would change. We only modify one assumption: time-invariant real wage is replaced by time-invariant real wage growth rate. The results are qualitatively valid but quantitatively different: the discounted total costs in terms of annual expenditures drop from 0.7 to 0.49 as the annual growth rate rises from 0 to 4%. Of course, the costs of the reform can be reduced by the elimination of Females40 but it is politically risky.

The structure of the paper is as follows. Section 2 presents the Hungarian statistics on the strange coexistence of rigid and loose systems. Sections 3 and 4 discuss the replacement of the rigid system with a flexible one for time-invariant and growing average real wages, respectively. Section 5 draws the conclusions. Appendices analyze complications like 1) Females40, 2) the welfare impact of a flexible system and 3) the experiences in Sweden, Czechia and Poland.

2 Rigid/loose retirement age in Hungary

In Hungary, between 2001 and 2007, the female FRA grew quite steeply, from 58 to 61 years but due to the weak incentives, the effective retirement age oscillated around 57.5 years. In 2009, FRA reached 62. while the effective retirement age jumped to 60 years.

Since 2011, there has been a radical change in pension policy: rigid and rising effective retirement age with large exceptions in Females40, Table 1 presents the statistics of the Hungarian old-age retirement system, 2011–2018. Notwithstanding the unification of male and female FRAs, the practice of Females40 requires the separation of the two genders, and within the females, favored and not-favored as well. As can bee seen, in each category, the average retirement age has been increasing, the size of the retiring cohorts has been oscillating. While in 2018, the FRA was already 63.5 years for those born in the second half of 1955, and the average (i.e. effective) male retirement age slightly surpassed it, the average Females40 retirement age was lower by 4 years and the total female average stayed at 61.2 years.

Table 1. Full and effective retirement ages and size of the retiring cohorts

	Full	Females40		Females Total		Males	
Year	Retirement age (yr)	Effective age (yr)	Size ('000)	Effective age (yr)	Size ('000)	Effective age (yr)	Size ('000)
2011	62.0	57.6	54.8	58.5	84.9	60.3	43.2
2012	62.0	57.8	26.6	59.2	51.0	61.8	21.0
2013	62.0	58.0	24.0	59.6	40.0	62.2	21.6
2014	62.5	58.3	27.5	59.6	38.9	62.8	18.9
2015	62.5	58.7	28.6	60.0	41.6	62.7	22.2
2016	63.0	59.0	28.3	61.1	55.9	63.1	22.6
2017	63.5	59.3	28.7	61.0	46.9	63.6	32.3
2018	63.5	59.6	29.0	61.2	49.6	63.7	35.0
2019	64.0	59.6	27.6	62.0	59.6	64.1	57.0

Based on Fazekas et al. (2020), Table 11.5, p. 240.

3 Model with time-invariant real wage

In our models, we assume that there is no inflation, the population is stationary, the life expectancy and the total fertility rate are time invariant, and each cohort is represented by a single person. In the basic model, we also apply an additional assumption: the (average) real wage is time-invariant. First we analyze a static (or steady state) model, then we turn to the dynamic. To avoid complications, we assume that certain events occur on December 31 of the corresponding year.

3.1 Static model

To prepare the ground, here we consider a static model (cf. Simonovits, 2003, Chapter 12). Our representative worker starts to work at age Q, works until age R, while earns w (including all pension contributions), her contribution rate is equal to $\tau \in (0,1)$ and after retiring, she dies at age D: $0 < Q < R < D$. Benefits in progress are equal to initial benefits. For later use, we introduce the number of contributive years: $S = R - Q$ and the duration in retirement: $T = D - R$. The benefit is equal to the ratio of the lifetime contributions to the remaining life expectancy:

$$b(R) = \frac{\tau S w}{T}, \qquad S = R - Q, \quad T = D - R. \tag{1}$$

In this system, the lifetime net contribution $z(R)$ is equal to 0. In fact, substituting $b(R)$ into

$$z(R) = \tau S w - b(R) T, \tag{2}$$

a simple calculation yields $z(R) = 0$. This is true, regardless of the variance of wage w, retirement age R and age at death D.

At the individual and governmental levels, however, the value of D is unknown, therefore we have to calculate with expected value in (1). From now on, we assume that Q and D are identical but R can vary within a narrow interval $R_{\mathrm{m}} \leq R \leq R_{\mathrm{M}}$.

In the pension literature it is customary to relate benefits to net wages, called replacement rate. As an illustration, we show how the benefit varies in the flexible system when the retirement age varies as $R = 60, \ldots, 70$, while $Q = 25$ and $D = 81$. Assuming a total wage compensation w and a contribution rate $\tau = 0.2$, hence a net wage $u = (1-\tau)w$, it is evident how low is the earliest net replacement rate and how high is the delayed one, both with respect to the normal one. In addition, we present the accumulated proportional modifications, which are strongly asymmetric.

Table 2. Retirement age, relative benefit

Retirement age (yr) R	Relative benefit $b(R)/u$	Accumulated modification $b(R)/b(65)-1$	Retirement age (yr) R	Relative benefit $b(R)/u$	Accumulated modification $b(R)/b(65)-1$
60	0.417	–0.333	66	0.683	0.093
61	0.450	–0.280	67	0.750	0.220
62	0.487	–0.221	68	0.827	0.323
63	0.528	–0.156	69	0.917	0.467
64	0.574	–0.075	70	1.023	0.636
65	0.625	0			

3.2 Dynamic model

We turn now to the simplest dynamic model. For the time being, we assume that each cohort is represented by one person and they start and end working plus die at the same age. The calendar years are denoted by $t = 0, 1, 2, \ldots$. To simplify the calculations, we need a complementary assumption: before the flexible system was introduced, the pension system had operated with a mandatory retirement age R for T years, and the system was in balance [(1)]:

$$Tb = \tau Sw, \qquad b = b(R). \tag{3}$$

(It would be more precise but too complicated to distinguish the rigid system's parameters as R^* etc.)

The government introduces a limited flexible system in year $t = 1$: the minimal retirement age is 1 year lower than FRA: $R_{\mathrm{m}} = R - 1$. The new cohorts retire at the minimal age: $R' = R - 1$. Then $S' = S - 1$ and $T' = T + 1$. Then the reduced benefit is equal to

$$b' = \frac{\tau(S-1)w}{T+1}. \tag{4}$$

What additional costs arise during the transition? For simplicity, we assume that the contribution rate remains invariant, and the temporary deficit is financed by the budget. We start with the determination of the first year's cost, which is the sum of the missing contribution of the early retirees and their added benefits, i.e. using (4)

$$Z_1 = \tau w + b' = \frac{S+T}{T+1}\tau w = \frac{1+\rho}{1+\rho S} S\tau w, \tag{5}$$

where $\rho = T/S$ denotes the old-age dependency ratio and $S\tau w$ is the original annual pension contribution (and expenditure).

We shall show that $(Z_t)_{t=1}^{T+1}$ is a finite arithmetic series. Using the formulas for old and new benefits (1) and (4), the change in the cost arising in year t is equal to

$$Z_t - Z_{t-1} = b' - b = \frac{S-1}{T+1}\tau w - \frac{S}{T}\tau w = -\frac{S+T}{T(T+1)}\tau w = \frac{1}{T}Z_1, \tag{6}$$

Then (6) and (5) yield

$$Z_t = Z_1 - (t-1)\Delta Z = Z_1\left(1 - \frac{t-1}{T}\right), \qquad t = 1, \ldots, T+1. \tag{7}$$

Finally, we calculate the total cost of transition:

$$C = \sum_{t=1}^{T} Z_t. \tag{8}$$

Inserting (5) and (7) into (8), and applying the formula for the sum of the arithmetic series:

$$C = \tau\frac{(S+T)w}{T}\frac{T}{2}\left(\frac{1}{T+1} + \frac{T}{T+1}\right) = \tau\frac{(S+T)w}{2}. \tag{9}$$

We shall relate this quantity to the original annual pension expenditure ($\tau S w$):

$$C = \frac{1+\rho}{2}\tau S w. \tag{10}$$

If $\rho = 0.4$, then by (5), the first year deficit is equal to $(1+0.4)(1+16) = 8.2\%$ of the annual expenditure, while by (10), the total cost of transition is equal to $(1+0.4)/2 = 0.7$ of the original annual expenditure.

It is evident that the starting as well as the transitory costs are significant. They would be even higher if we allowed for earlier retirement, 2 or 3 years below FRA and if the workers chose them. Here we present a simple estimate for a 2-year reduction. Calculating with double years, $R'' = 63$, then $S'' = 20$ and $T'' = 8$, $w'' = 2w$, i.e. $C'' = 2C$, the total costs would be double of the 1-year reduction.

Of course, in reality, not everybody retires as soon as possible; there are even workers who delay retirement beyond FRA. But to open the window of opportunity down is much easier than to open it up. For example, if the widths of the window are equal to 3 years, then everybody can retire at 62. But those, who wanted to retire at 68 but were forced to retire at 65, cannot return to work. The opening the window in both directions requires time. It is conceivable that the opening should be step-wise: $R_{\mathrm{m}}(2023) = 64$, $R_{\mathrm{m}}(2024) = 63$ and $R_{\mathrm{m}}(2025) = 62$ years.

We note that it is possible to consider a contribution rate which balances the pension system every year but it would complicate (1), namely in its numerator, τS would be replaced by $\tau_{t+1} + \cdots + \tau_{t+S}$. We also neglected the variance of individual life expectancies. Furthermore, workers die between minimum and maximum retirement ages, therefore the expected remaining life expectancy $D_R - R$ is higher than our $D - R$, but we skip this complication, too. Our model would be even more complex if we took into account the fragmentation of the careers, the variances of earnings of the remaining life expectancies, and of the cohort sizes. Here we would need serious empirical research. I can only cite Péter Vékás' information: in Hungary, the remaining life expectancy at 60 is about 20 years, and its standard deviation is almost 10 years.

Due to historical reasons, in Hungary the accrual rate—which determines the benefit as a function of the length of contributions—is increasing but its steepness hectically depends on the very length. As a result, the aggregate accrual rates are equal to 53 vs. 27% for the first and the second 20 years, respectively. This anomaly is to be removed (as was already planned for 2009) and regardless of the introduction of the flexible system, it should be replaced by a constant rate. The delayed retirement credit and the deduction rate for early retirement both might be equal to 6%/year, in addition to the marginal accrual rate of 2%.

4 Model with increasing real wage

Relaxing our very strong assumptions the quantitative results would change. In this paper, we only relax one assumption: time-invariant real wage is replaced by time-invariant real wage growth rate. The results qualitatively remain valid. Reflecting the Hungarian specifics, the cohorts' benefits in progress remain age-invariant in real terms but the initial benefits and the real wages increase with time. We shall see that our cost estimates basically remain correct.

Again, we start the analysis with the quasi stationary version: $R_t = R$, and only after finishing it we shall turn to the dynamic model: $R' = R - 1$.

4.1 Quasi-stationary model

We shall need the real wage dynamic:

$$w_t = w_0 g^t, \tag{11}$$

where g stands for the real growth coefficient (=1+growth rate). We shall see (cf. Simonovits, 2020) that due to the economy of indexing to prices, the initial pension should be corrected upward, by the imputed contribution rate τ_g:

$$b_t = \frac{\tau_g S w_t}{T}. \tag{12}$$

Then the cross-sectional balance condition in year t is as follows:

$$\tau S w_t = \sum_{i=0}^{T-1} b_{t-i}. \tag{13}$$

Inserting (11)–(12) into (13):

$$\tau S w_t = \frac{\tau_g S w_t}{T} \sum_{i=0}^{T-1} g^{-i}. \tag{14}$$

Simplifying (14) and using the formula for the sum of the geometric series imply

$$\tau_g = \tau T \frac{1 - g^{-1}}{1 - g^{-T}}. \tag{15}$$

Introducing notation

$$T_g = \frac{1 - g^{-T}}{1 - g^{-1}}$$

and inserting (15) into (12) yield

$$b_t = \frac{\tau S w_t}{T_g}. \tag{16}$$

Comparing (16) to (1), one can see that T_g stands for the "reduced duration in retirement" due to price indexation. Question: how can the government guess the future real wage growth rate ahead? Answer: the government can only forecast it and rely on buffer stocks in corrections (cf. Sweden).

4.2 Dynamic model

Repeating the introduction of Subsection 3.2, we analyze the scenario of reducing the retirement age while the real wages increase. We assume again that between years –15 and 0, a mandatory retirement age of 65 was operating, the real wages grew exponentially and the benefits in progress kept their real values. What happens during the transition? Generalizing the story of the previous Section, we still assume that the contribution rate remains constant and the transitional deficit is financed by the government.

Again, from year $t = 1$, flexible retirement is introduced: $R_{\mathrm{m}} = R - 1$. In an extreme case, everybody retires at the minimum age: $R' = R-1$. Then $S' = S-1$ and $T' = T+1$. We present the initial benefits but now with rising real wages:

$$b_0 = \frac{\tau w_0 S}{T_g} \qquad \text{and} \qquad b_t = \frac{\tau w_t (S-1)}{T_g + g^{-T}} = b_1 g^{t-1}, \qquad t = 1, \dots, T+1. \tag{4′}$$

It is worth directly calculating the first year's deficit, which is the sum of the missing contributions of the freed cohort and the newly awarded benefits:

$$Z_1 = \tau g w_0 + \tau \frac{S-1}{T_g + g^{-T}} w_0 g. \tag{5′}$$

To find the general solution, we need the time-varying average benefits (cf. (14)):

$$\bar{b}_t = \frac{b_1(1 + \cdots + g^{t-1}) + b_0(1 + \cdots + g^{-T+t})}{T+1}, \qquad t = 1, \dots, T$$

and $\bar{b}_{T+1} = \tau(S-1)w_0 g^{T+1}/(T+1)$.

The costs arising during the transition are given by the generalizations of the (6) and (7). We also display the cumulated costs as well:

$$C_t = \sum_{s=1}^{t} Z_s. \tag{17}$$

Numerical example: It would be useless to the derive analytical formulae, therefore we present the numerical paths of the initial benefits in terms of the total wage compensation in year 0: $w_0 = 1$, the annual and the accumulated costs, in terms of the original pension expenditures B_0. $R = 65$, $R' = 64$ year, $N = 100\,000$ persons, $g = 1.02$. It can be seen that due to rising real wages, the relative average pensions are rising, the relative annual costs converge to zero and the accumulated costs converge to 66%, lower than in the static case.

Table 3. Average benefits, annual and cumulated costs (rising real wages)

Year t	Gross average benefit $\bar{b}_t$	Annual cost/Original expenditure Z_t/B_0	Cumulated costs/Original expenditure C_t/B_0
0	0.500	–	–
1	0.503	0.074	0.074
2	0.510	0.070	0.143
3	0.518	0.066	0.209
4	0.526	0.062	0.271
5	0.534	0.057	0.328
6	0.542	0.053	0.381
7	0.550	0.049	0.430
8	0.558	0.044	0.474
9	0.567	0.040	0.514
10	0.576	0.035	0.549
11	0.585	0.030	0.580
12	0.594	0.026	0.605
13	0.603	0.021	0.626
14	0.613	0.016	0.642
15	0.622	0.011	0.652
16	0.632	0.005	0.658
17	0.642	0	0.658

To see the sensitivity of our calculations just to the growth rate of the real wages, Table 4 presents the initial average benefits and costs and the accumulated cost, each in terms of the initial total wage for various rates. In addition to the undiscounted accumulated cost, we shall calculate the discounted one, where the discount factor is equal to the growth factor:

$$C_t^g = \sum_{s=1}^{t} Z_s g^{-s}. \tag{17'}$$

Calculating with an annual growth rate 4%, the initial benefit slightly grows, the initial, the cumulated and the discounted costs are decreasing by 20, 12 and 30%, respectively.

Table 4. Benefits, starting and cumulated costs for various growth rates

Growth coefficient of real wage g	Average gross benefit $\bar{b}_1$	Annual cost/Original expenditure Z_1/B_0	Cumulated costs/Original expenditure C_t/B_0	Discounted costs/Original expenditure C_t^g/B_0
1.00	0.498	0.082	0.700	0.700
1.01	0.500	0.078	0.679	0.639
1.02	0.503	0.074	0.658	0.584
1.03	0.505	0.070	0.637	0.534
1.04	0.508	0.066	0.617	0.488

5 Conclusions

In mature market economies, the flexible retirement harmonizes efficiency and fairness. For example, in the US, Germany and Sweden, it functions without any special limitation. The same system works in several ex-socialist economies, but with strong limits. For example, in Czechia, one has to have to contribute at least 35 years to retire before reaching FRA and the deduction is severe. These limitations seem to be excessive and judged so by OECD (2020, p. 26). In Slovakia, the actuarially reduced initial benefit has to reach the minimum wage, which is close to the average benefit. One Polish government raised both FRAs then another renounced the long-term equalization of female and male FRAs and reduced them to the original values.

In Hungary, the early retirement rules applied too low deductions before 2010. For example, between 2003 and 2007, the female FRA rose from 59 to 61, but the effective retirement age remained the same: cc. 57.5. Therefore it was correct to raise the deductions in 2010. Its elimination from 2012, however, was a clear mistake. After a decade freeze, it is worth introducing an actuarially fair retirement system, but this involves non-negligible temporary costs for two decades. The cost can be diminished by phasing out the otherwise popular Females40, but it is quite demanding politically.

These strange combinations of loose and rigid retirement age policies are far from being socially optimal but their replacement with a truly flexible system looks politically difficult. In my opinion, all the four rigid systems should be replaced but its transitory costs should be be taken into account.

In contrast to previous studies (e.g. Fabel, 1994; Diamond, 2003; Eső and Simonovits, 2002), now we do not study how the government chooses the pension rules and how the workers react to them. Following others, the current paper neglects the impact of indexation on retirement decision. Simonovits (2019) and (2020) suggest that with respect to price indexation, the otherwise superior wage indexation weakens the incentives to delay retirement. I only aimed to extend the static analysis to dynamic.

I tried to estimate these costs in a simplest model. Better models can involve more realistic assumptions, extending its validity to other, partially flexible retirement systems as well. The decision to introduce early retirement with actuarially fair deductions has to be made by the corresponding governments.

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Appendix 1. Estimation of the cost of Females40

We have already mentioned that Females40 is not only unfair but costly. Official data of 2019 put its cost to 6% of the total pension expenditure. This number, however, is debatable, because it mechanically calculates the total benefits paid to the beneficiaries of the program. The true costs are different and this Appendix tries to estimate them. We need the following notations: The program started in year $U_1 = 2011$ and will terminate in year U_2 meaning that no new members are accepted. Lower index t refers to the year of retirement. N_t^{o} is the number of retirees in Females40, R_t^{o} is their average retirement age, w_t stands for the total wage compensation. We calculate with three types of costs (i) missing contribution, (ii) the early benefits, and (iii) the excessive benefits paid after the FRA. We assume that few Females40 retirees die before the closure of the program and we do not model post closure events.

Ad (i) The missing contributions in year t. Roughly 4 years' contribution is missing, those retiring within Females40 in years in $t-3, t-2, t-1, t$:

$$V_t^1 = \tau_t(N_{t-3}^{\mathrm{o}} + N_{t-2}^{\mathrm{o}} + N_{t-1}^{\mathrm{o}} + N_t^{\mathrm{o}})w_t, \qquad t = U_1, \ldots, U_2.$$

Here $N_{U_1-3}^{\mathrm{o}} = N_{U_1-2}^{\mathrm{o}} = N_{U_1-1}^{\mathrm{o}} = 0$, though there were other similar programs before 2011.

Ad (ii) The direct cost of early retirement in year t:

$$V_t^2 = N_{t-3}^{\mathrm{o}} b_{t-3} + N_{t-2}^{\mathrm{o}} b_{t-2} + N_{t-1}^{\mathrm{o}} b_{t-1} + N_t^{\mathrm{o}} b_t, \qquad t = U_1, \ldots, U_2.$$

Ad (iii) The additional benefits of Females40 (after R_t^*)

$$V_t^3 = V_{t-1}^3 + N_t^{\mathrm{o}}(\Delta b_t), \qquad t = U_1, \ldots, U_2,$$

with initial value $V_{U_1-1}^3 = 0$. On the one hand, the difference Δb_t between the unchanged and the reduced benefit is a burden for life. On the other hand, the price indexation and the recent real wage explosion reversed the process: a large part of the Females40 would have gained if they had delayed their retirement, but they did not know about it and the government failed to warn them.

Appendix 2. The welfare provided by flexible retirement

In the main part of the paper we have avoided the use of utility functions, but to analyze the welfare provided by flexible retirement we need utilities. In Appendix 2, constant real wages are assumed. We choose the simplest utility function:

$$U[R] = (R-Q)[\log((1-\tau)w) - \theta] - \varphi(R - R_{\mathrm{m}})_+ + (D-R)\log b(R), \qquad (A.1)$$

where parameters θ and φ represent the labor disutility of one year work during the whole interval $[Q, D]$ and the final interval $[R_{\mathrm{m}}, D]$, respectively; the subindex + refers to the positive part of a real and $b(R)$ is defined in (1). (Implicitly, we fixed the contribution rate τ, because there is a wide-spread opinion that it should not be raised further.) Evidently, the greater the θ and φ, the earlier the worker wants to retire. Since the numerical value of the utility function has no direct economic meaning, we shall rely on the *relative efficiency*, defined as follows. This is a real number ε, by which multiplying both the wage and the benefit, the rigid system (R^*) provides the same utility as the flexible system (R) without multiplication. (Here we follow the precise notation, which we mentioned below (3), forsook until now.) In formula: using notation

$$U(R, \varepsilon) = (R-Q)[\log((1-\tau)w\varepsilon) - \theta] - \varphi(R - R_{\mathrm{m}})_+ + (D-R)\log(b(R)\varepsilon) \qquad (A.2)$$

the relative efficiency satisfies the following implicit equation:

$$U(R^*, \varepsilon) = U(R, 1). \qquad (A.3)$$

Using (A.2), a simple calculation yields

$$U(R, \varepsilon) = U(R, 1) + (D-Q)\log\varepsilon,$$

i.e. (A.3) implies

$$U(R^*, 1) + (D-Q)\log\varepsilon = U(R, 1).$$

Hence the explicit formula for the relative efficiency is

$$\varepsilon = \exp\{[U(R,1) - U(R^*,1)]/(D-Q)\}.$$

Adopting the numbers of Table 2, Table A1 shows the relative efficiency for $\varphi = 0.1$ with various θs. In column 3, $\theta = 2$, characterizing those workers with high labor disutility, who gain with early retirement. As can be seen, the optimal retirement age (italicized) $R^{o} = 63$ raises utility by 0.4% above that of the rigid 65 year, but the real loss of utility arises with higher age. In column 4, $\theta = 1.8$ represents those workers, with medium labor disutility for whom the optimal retirement age is just equal to the FRA. In column 5, $\theta = 1.5$ stands for those workers, with low labor disutility, who gain with the delay: $R^{o} = 67$ yields welfare 0.5% higher than the rigid 65, but the real loss of welfare arises for forced early retirement.

Table Aú1. The relative efficiency for early, normal and late retirement

Effective retirement age R	Benefit/Wage b/w	Labor disutility: high ε_1	medium ε_2	low ε_3
62	0.389	1.003	0.992	0.976
63	0.422	*1.004*	0.997	0.986
64	0.459	1.003	0.999	0.994
65	0.500	1.000	*1.000*	1.000
66	0.547	0.995	0.998	1.004
67	0.600	0.987	0.995	*1.005*
68	0.662	0.978	0.988	1.004

Appendix 3. Retirement ages in Sweden, Czechia and Poland

In this Appendix we review the experiences with flexible retirement system in Sweden, Czechia and Poland.

Palmer and Könberg (2020, pp. 30–32) described the Swedish system as satisfactory. The starting point is the pioneer NDC, which almost directly implies flexibility. There is no need for FRA but for limiting the use of supplementary pension, nobody can rely on it below 65. There is an earliest retirement age: 61 years, below which only disability pension is available. There is another, upper limit: 67 years, where the employer can freely fire the worker. At the end of 2017, the six democratic parties agreed to raise the retirement ages: to raise the minimal retirement age from 61 to 64 years between 2020 and 2026, in three steps; the threshold of supplementary benefit, 65 is raised to 66 years; and the threshold of free firing from 67 to 69, both in two steps. The fast rise of the minimal age, however, may represent dissatisfaction with the stagnating effective retirement age.

At the start of the reform (cc. 2000) 90% of the Swedes retired before or at 65 years; this value was still 80% but the average remained at 65. The 1950-cohort's 28% retired before 65, 50% at age 65 and 22% after 65. In a certain degree, the choice of the retirement age is rational. For example, workers with lower skill, whose life expectancies are lower, retire as early as possible.

The following question arises: to what extent does the DC system influence the retirement decision? As a background information, we note that between 1970 and 1990, the effective retirement age sank all over the world. The Swedish reform contributed to the reversion of the process.

OECD (2020) critically analyzed the Czech pension system. Here we only confine our attention to the timing of the retirement as a function of the length of contributions. The Czech system is flexible but the conditions are very stringent: the worker has to work at least 35 years claiming pension before reaching the FRA, and the reduction is very severe (e.g. for retiring 5 years earlier, the reduction is 43% in contrast to –33.3%). Figure 1.12 (OECD, 2020, p. 27) shows the distributions of retirees according to years of contribution: Figure A for FRA, Figure B for early retirees. Note that both distributions are concentrated at very high values!

Figure A. The distribution of new retirees with respect to years of contribution, FRA

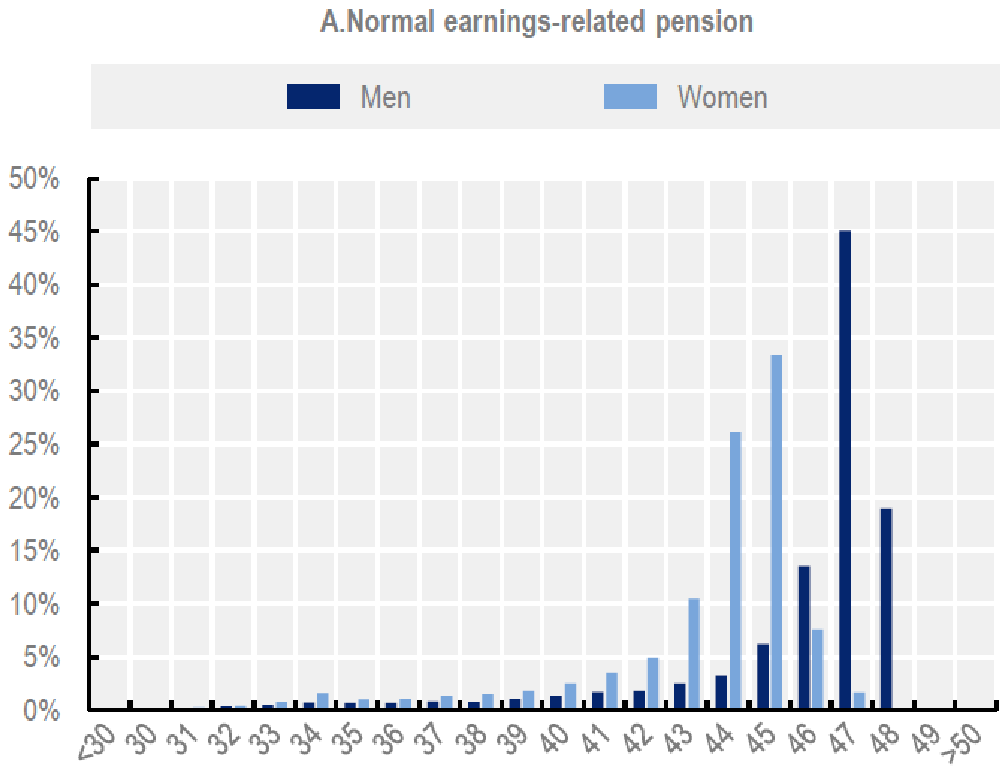
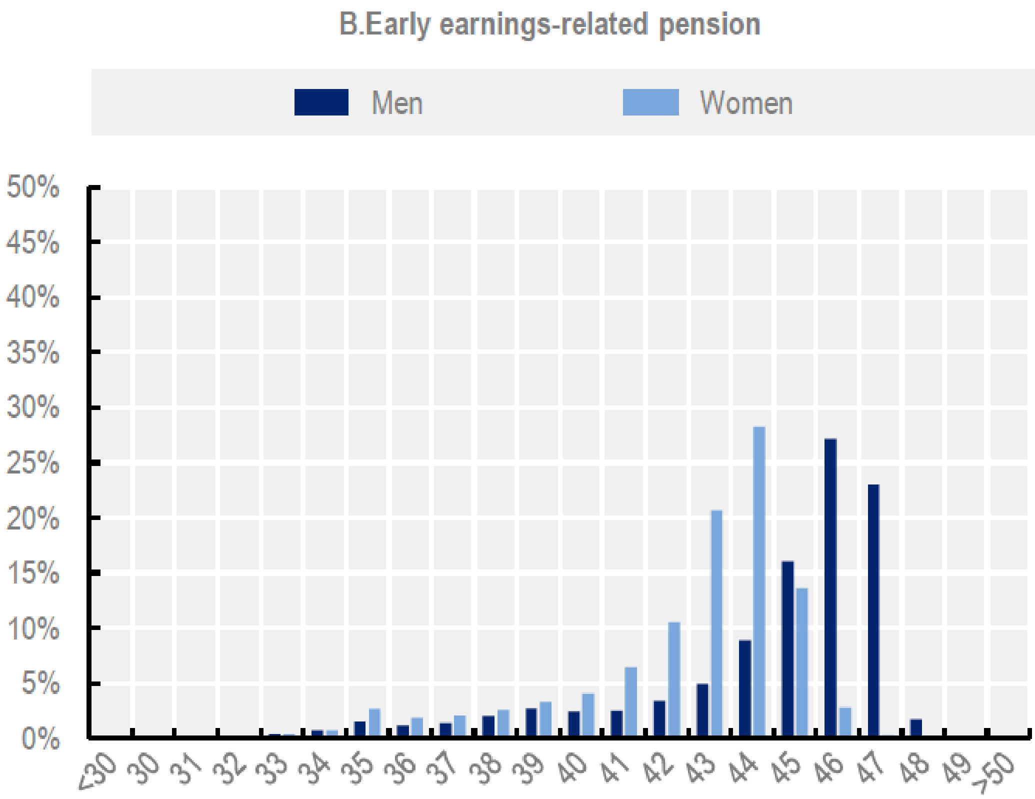


Figure B. The distribution of new retirees with respect to years of contribution, early



Turning to Poland, this country has had rather high male and female FRAs since 1954: 65 for males and 60 for females, respectively. The earliest retirement ages, however, were 5 years lower and the actual ones even lower. The relatively low life expectancies had been stagnating until the 1990s. Due to the mass unemployment during the transition period, effective retirement ages dropped before started increasing. As late as 2012, male effective retirement age was as low as 59.7 years, and in 2007 the female counterpart 53.8 years. Since then, both effective retirement ages have been increasing, to 64.6 and 61.0, respectively, stabilizing the retirement duration.

Table A2. Peaks and troughs in effective male retirement ages and durations, Poland

Year	Effective retirement age	Retirement duration
1990	58.5	16.0
1997	58.3	17.4
2003	60.5	17.1
2010	62.0	16.2
2013	59.7	18.7
2017	64.6	15.9

Table A3. Peaks and troughs in effective female retirement ages and durations, Poland

Year	Effective retirement age	Retirement duration
1990	55.8	23.3
1997	54.1	25.9
1999	56.7	23.7
2007	53.8	28.0
2011	59.5	23.8
2017	61.0	23.5

Source: Chłoń-Domińczak (2019, Figure 2).