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Indexing public pensions in progress to wages or prices

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ABSTRACT

Initial public pensions are indexed to the economy-wide average wages, but pensions in progress are indexed to prices, average wages or their combinations––varying across countries and periods. We create a simple overlapping cohorts framework to study the properties of indexing pensions in progress––emphasizing a neglected issue: close wage paths should imply close benefit paths even at real wage shocks. This robustness criterion of an equitable pension system is only satisfied by wage indexing, which in turn requires the adjustment of the accrual rate. To minimize the redistribution from low-earning short-lived citizens to high-earning long-lived ones, progression should be introduced.

JEL codes: D10, H55

Keywords: public pensions, indexation, horizontal equity, pensions in progress

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A már megállapított nyugdíjak bérek vagy árak szerinti indexálása

SIMONOVITS ANDRÁS

ÖSSZEFOGLALÓ

A már megállapított nyugdíjak bérek vagy árak szerinti indexálása

A kezdőnyugdíjakat általában az átlagos béremelkedés ütemével valorizálják, de a már megállapított nyugdíjak indexálása térben és időben váltakozva, követheti a béreket, az árakat és kombinációjukat. Egy egyszerű együtt élő évjárati modellt készítünk, amelyben jól vizsgálhatjuk a már megállapított nyugdíjak indexálását, hangsúlyozva egy elhanyagolt kérdést: közeli kereseti pályáknak közeli nyugdíj-pályákat kell adniuk, még jelentős reálbér-sokkok esetén is. Ezt a robusztussági ismérvet csak a bérindexálás elégíti ki, de ez a járadékszorzó megfelelő csökkentését igényli. Ha minimalizálni akarjuk jövedelem-újraelosztást a rövid várható élettartamú, alacsony keresetű állampolgároktól a hosszú várható élettartamú, magas keresetűek felé, akkor degressziót kell alkalmaznunk a kezdőnyugdíjaknál.

JEL: D10, H55

Kulcsszavak: tb-nyugdíjak, indexálás, horizontális méltányosság, már megállapított nyugdíjak

Indexing public pensions in progress to wages or prices*

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Abstract

Initial public pensions are indexed to the economy-wide average wages, but pensions in progress are indexed to prices, average wages or their combinations—varying across countries and periods. We create a simple overlapping cohorts framework to study the properties of indexing pensions in progress—emphasizing a neglected issue: close wage paths should imply close benefit paths even at real wage shocks. This robustness criterion of an equitable pension system is only satisfied by wage indexing, which in turn requires the adjustment of the accrual rate. To minimize the redistribution from low-earning short-lived citizens to high-earning long-lived ones, progression should be introduced.

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1. Introduction

Since the 1970s, almost all over the developed world, *initial public pensions* have been indexed (valorized) to the economy-wide average wages, but *pensions in progress* have been indexed to prices, average wages and various combination of prices and average wages—varying across countries and periods. Indexing pensions is only a technical measure in the short run, but it can be very important in the long run. This is especially true when the public pension (paid as unisex indexed life annuity) replaces a large part of the previous earnings. In my opinion, in the pension literature, indexing pensions has not received the attention which it deserves. Perhaps it is not an accident that in his excellent AEA Presidential lecture, Peter Diamond had to relegate the problem into a footnote (Diamond 2004, p. 7. ftn. 24):

"Mandatory annuitizition in a social security program raises the interesting question of how a monthly benefit should vary over time—with prices, wages, and possibly other variables such as rates of return. Relevant for this issue are the age structure of optimized expenditures, the relative importance of both real and relative consumption, and the allocation of risk bearing between the elderly and the rest of population. Currently [in the US in 2004, A.S.], the benefits in force are increased for inflation as measured by the CPI. While this is a reasonable solution, I suspect it would be better, on a revenue neutral basis, to have lower initial benefits that then grew faster (for example as a weighted average of prices and wages). This would help more the longer-lived than the shorter-lived but the effect on expected lifetime income distribution could be partially adjusted by changing the benefit formula."

Barr and Diamond (2008) devoted a whole chapter (Chapter 5) to this multidimensional problem: they separately discussed indexing initial and continued benefits by dividing the pension period into two subperiods, answering some issues raised by Diamond (2004). Very few countries use and very few economists favor indexing initial pensions to prices (namely, President's Commission, 2001, p. 84, model 3 and Biggs, Brown and Springstead, 2005). On the other hand, both practices and opinions are divided whether pensions in progress should be indexed to wages or prices or their combination. Therefore we confine our attention to indexing benefits in progress and study the problem with a multicohort rather than a two-generation overlapping model. Apart from Theorem A.1 in Appendix A, in this paper we avoid combined indexation, and suggest that indexation to wages (similar to a point system—see Appendix A—or Nonfinancial Defined Contribution) is superior to indexation to prices, especially when a real wage shock hits the economy.

In the short run, in a country with smooth real wage dynamics, the method of indexing pensions in progress is almost irrelevant. With an annual consumer price index of 102 and a nominal wage index of 104, at first sight it is not too interesting if the nominal pensions are increased by 2 or 4%, or their arithmetic average, by 3%. A typical pensioner, however, spends about 20 years in retirement, therefore the annual 1–2% differences become 20–40% differences at the end and 10–20% deviations during the whole period. The latter difference manifests itself both at the macro and the micro levels. But even in the well-designed US Social Security system, the correction of an earlier indexation error created so-called *notch babies*: cohorts retiring just after 1977 and born after 1916 received much lower benefits than slightly earlier cohorts (e.g. Krueger and Pischke, 1992).

In another country where average real wages may increase or decrease by 5–10% a year, with a relative freedom from the also fluctuating GDP's growth rate, indexation matters even in the short-run. For example, Table B.1 in Appendix B shows turbulent real net wage and benefit dynamics in Hungary. To focus on two events: those Hungarians, who retired in 2017, 2018 or 2019 received pensions in real terms higher by 7, 18 and 28% (cumulated real growth rates of wages, respectively) than those, who—with similar wage paths—retired in 2016. In contrast, price-wage indexation plus extra measures preserved robustness in 2001–2003, when real wages also grew quickly, close wage paths implied close benefit paths.

As Table B.2 shows, real net wage hikes have not been limited to Hungary; in a number of other countries, growth rates of real net wages wildly oscillated. Even between 2014 and 2017, cumulated wage hikes in Latvia and Lithuania amounted to 20.8 and 17.7%, respectively. The impact of wage turbulence on benefits in the foregoing and similar countries needs further inquiries. A very recent example: in July 2019, the Romanian parliament enacted a 60% raise of the average pensions by 2020! Therefore indexing pensions deserves the attention of both theoretical and applied economists.

The main message of the paper is as follows: the only method to achieve robustness is to raise pensions in progress by the economy-wide wage growth rate, shortly: *indexing to wages*. (The German point system and the Swedish NDC take into account population aging, too.) At the same time, this type of indexation makes the necessary restraint with the initial benefits more visible and prefers those living longer (females and higher earners); moreover, it weakens the incentives to retire later. Finally, in a country, where the pension system is proportional (equivalently: earnings-related) and benefits in progress are indexed to wages, some form of a flat component is inevitable. At this point I must admit that between 2010 and 2017 I also accepted the prevailing wisdom in Hungary: only pure price indexing is politically feasible in the long run. The preservation of combined indexation or the return to pure wage indexation would have required the simultaneous reduction of accrual rate (which connects the lifetime wages to the initial benefit)—deemed unrealistic then!

In this paper, we use the framework of overlapping cohorts with a stationary population. In the macrosections, each cohort is represented by a single individual, whose real net earnings vary with years. Under indexation to wages, perhaps after a transition period, the benefits become independent of the age of the beneficiary and follow the wage dynamics through the (total) accrual rate.

What happens under the more popular indexation to prices? Referring to the ratio of the average benefits to the average wages as the *average replacement ratio* (sometimes called benefit ratio), we show that the higher the time-invariant real net wage growth rate, the lower the average replacement ratio. We mention two consequences: (i) low average replacement ratio implies relative poverty among the pensioners and (ii) lack of robustness: a real net wage hike, separating two subsequent cohorts, may transform two close wage paths (differing only in the start and the end years) into distant benefit paths.

Returning to the study of wage indexing, note that the representative individuals can be simply replaced by multi-type cohorts with varying wages, life expectancies and fragmented labor careers but by assuming time-invariant and type-invariant growth rates of real wages. (Arbitrary wage paths are relegated to Appendix A.)

A short review of the relevant literature follows. Feldstein (1990) created a special model of two overlapping generations of *pensioners* (plus a third generation of workers) and studied the socially optimal age structure of the US Social Security benefits—regardless of any social custom. Simonovits (2003, Section 14.4) modeled the intercohort impact of replacing wage indexing by price indexing in an annual rather than a decade model of Feldstein (1990) or Barr and Diamond (2008). Legros (2006) analyzed the interaction of indexation and lifetime redistribution. Lovell (2009) dissected the inconsistencies in the US Social Security rules.

Perhaps Auerbach and Lee (2011) is closest to the target of our study. They created a stochastic simulation model to analyze how public pension structures spread the risks arising from demographic and economic shocks across generations. Starting from the qualitative features of the US, the Swedish and the German public systems, they compare various sustainable systems. “Using a horizontal equity index, [they] also compare the different systems’ performance in terms of how neighboring generations are treated” (Auerbach and Lee, 2011, p. 16). Nevertheless, the two models are utterly different. Theirs is a very sophisticated model, studying the long-run stochastic behavior of the pension system, considering very slow average real wage growth (of 1%/year). Ours is a very rudimentary model, concentrating on the real wage shocks (of order 10%/year) occasionally hitting certain emerging economies.

Weinzierl (2014) analyzed the impact of various price indices on the US Social Security system. Jaravel (2019, p. 715) demonstrated that “in the United States from 2004 to 2015... annual inflation for retail products was 0.661 ... percentage points higher for the bottom income quintile relative to the top income quintile.” Knell (2018) gave a deep critique of various versions of cohort-specific NDC rules (Holzmann and Palmer, eds. 2006) with rising life expectancy. We have to underline that for technical reason, we skip the very serious threat to any pension system, namely the population aging, resulting from the simultaneous rise in life expectancy and fall in fertility (below the critical value of 2.1).

At this point, we have to consider the issue of lifetime redistribution in the pension system. Since 2000, several economists have documented that the apparently progressive US Social Security system (with steeply declining marginal accrual rates) is only weakly progressive on a lifetime basis, because life expectancy at retirement is a steeply increasing function of the lifetime wages (e.g. Liebmann, 2002). Recently there is a growing concern for this tendency which is strengthening all over the world. Among others, Whitehouse and Zaidi (2008), The National Academy ... (2015), Chetty, Stepner and Abraham (2016), Auerbach et al. (2017); Ayuso, Bravo and Holzmann (2017) reconsidered this problem on newer data. Simonovits (2018, Section 14.4) returned to Diamond’s concern: the impact of *wage index weight* (i.e. the share of the wage index in the combined wage–price index) on the redistribution from the short-lived low-paid to the long-lived high-paid. Palmer and Zhao (2019) surveyed various issues of calculating life expectancy and indexing pensions in progress including its interaction with income-dependent life expectancy.

It is hardly discussed in the literature that the type of indexation also influences the choice of the retirement age. As an outlier, Simonovits (2019) modeled Female40, a seniority retirement system (in force in Hungary since 2011) which allowed females with eligibility of 40 years to retire without actuarial deduction. From the start this system unduly punished females with slightly shorter careers. Moreover, it became a boomerang due to price indexing and real net wage hike: since 2016, a large share of new beneficiaries would have received greater lifetime benefits if they had retired later. This is the case where two wrong incentives, namely Female40 and excessive real wage rise counteract.

Schookkaert, Devolder, Hindriks and Vandbroucke (2018) discussed a related model of the point system which is equitable and sustainable. Their model is more elaborate than ours, especially that it contains a general demographic block and a sophisticated blend of defined benefit (DB) and defined contribution (DC) principles for Belgium. But the foregoing model neglects a basic concern of the present paper: fragmented careers (Augusztinovics and Köllő, 2008; Simonovits; 2018, Section 9.4). Future research should combine the two approaches.

OECD (2019) gave a critical survey on the Hungarian pension policy and has formulated interesting proposals on indexation of initial and continued benefits. Figure 1.2 (on p. 84) demonstrated that using a 3- or a 10-year moving average of economy-wide wages in valorization would smooth the wild fluctuations in the real values of the Hungarian initial benefits. Figure 1.16 (on p. 99) showed the widening gap between expenditures under indexation to wages and prices starting in 2017, ending in a 3%point difference in terms of GDP in 2070 in a typical country like Hungary.

The structure of the present paper is as follows. Sections 2 and 3 discuss the wage and price indexing rules at a macrolevel, respectively. Section 4 generalizes the wage indexing to multi types and Section 5 concludes. Appendix A considers the point system for individual real net wage paths differing from the average path and combined indexation. Appendix B displays selected statistics on real wage and benefit dynamics.

2. Indexing to wages (macro)

All wages and benefits will be calculated at constant prices. We work with a very simple dynamic framework of overlapping cohorts with stationary population; especially simple in Sections 2 and 3, where each cohort will be represented by a single person. The representative person works S years and then spends T years in retirement, S and T are positive integers, close to 40 and 20, respectively. While working, the real net wage is independent of her age but depends on the calendar year.

We shall make the simplest assumption on benefits. The *initial* pension benefit is proportional to the current net wage v_t:

$$b_t = \beta v_t, \qquad t = 1, 2, \ldots, \tag{1}$$

where β is called the total *accrual rate* (or quite confusingly, replacement ratio—because under certain conditions, it shows the replacement of the last wage by the first benefit). For the sake of utmost simplicity, in (1) we neglect the usual one-year-lag in valorization. We shall derive the general formula for arbitrary individual wage paths in Appendix A. We shall also show that (1) is not only a final pay scheme but a good macro approximation of indexation of initial benefits.

In this Section, the benefits *in progress* are indexed to wages, i.e. every year the government raises these benefits according to the rationally expected time-variant net wage growth coefficient $g_t = v_t/v_{t-1}$. We have then

Theorem 1. *Under wage indexing, the initial benefit and—regardless of the years elapsing since retirement—the benefits in progress are equal to each other and are proportional to the current net wage: (1).*

Proof. Consider first the worker who retired in year $t-1$, her initial benefit was equal to $b_{t-1} = \beta v_{t-1}$. Due to indexation to wages, in year t, her resulting benefit in progress is equal to $g_t \beta v_{t-1} = \beta g_t v_{t-1} = \beta v_t = b_t$. By mathematical induction, the same applies to workers who retired $2, \ldots, T-1$ years before t. ∎

To highlight the impact of indexation on robustness, we create Table 1. The left half of Table 1 displays the life paths of two cohorts starting to work in years 0 and 1, respectively; under wage indexing (the right half will be used in Section 3). Their wage and benefit paths only differ at the start and the end. The differences arise in cohort 0's first wage v_0 and benefit $b_S^{\mathrm{v}} = \beta^{\mathrm{v}} v_S$ and in cohort 1's last wage v_S and benefit $b_{S+T}^{\mathrm{v}} = \beta^{\mathrm{v}} v_{S+1}$, otherwise the corresponding wages and benefits are equal. (To avoid confusion, here we distinguish the variables of wage- and price-indexed systems by superscripts v and p, respectively; but otherwise we may drop the superscripts.)

Table 1. *Wages, pensions indexed to wages vs. prices: shifted paths*

Year	Indexation to wages		Indexation to prices	
	Start at 0	Start at 1	Start at 0	Start at 1
t	$v_t \mid b_t^{\mathrm{v}}(0)$	$v_t \mid b_t^{\mathrm{v}}(1)$	$v_t \mid b_t^{\mathrm{p}}(0)$	$v_t \mid b_t^{\mathrm{p}}(1)$
0	v_0	–	v_0	–
1	v_1	v_1	v_1	v_1
...		...		...
$S-1$	v_{S-1}	v_{S-1}	v_{S-1}	v_{S-1}
S	$\beta^{\mathrm{v}} v_S$	v_S	$\beta^{\mathrm{p}} v_{S-1}$	v_S
$S+1$	$\beta^{\mathrm{v}} v_{S+1}$	$\beta^{\mathrm{v}} v_{S+1}$	$\beta^{\mathrm{p}} v_{S-1}$	$\beta^{\mathrm{p}} v_S$
...		...		...
$S+T-1$	$\beta^{\mathrm{v}} v_{S+T-1}$	$\beta^{\mathrm{v}} v_{S+T-1}$	$\beta^{\mathrm{p}} v_{S-1}$	$\beta^{\mathrm{p}} v_S$
$S+T$	–	$\beta^{\mathrm{v}} v_{S+T}$	–	$\beta^{\mathrm{p}} v_S$

Next we calculate the undiscounted lifetime benefits of the two subsequent cohorts introduced in Table 1:

$$C_0^{\mathrm{v}} = \sum_{t=0}^{T-1} b_{S+t}^{\mathrm{v}} \quad \text{and} \quad C_1^{\mathrm{v}} = \sum_{t=1}^{T} b_{S+t}^{\mathrm{v}},$$

hence their difference is equal to

$$C_1^{\mathrm{v}} - C_0^{\mathrm{v}} = b_{S+T}^{\mathrm{v}} - b_S^{\mathrm{v}} = \beta^{\mathrm{v}}(v_{S+T} - v_S) = \beta^{\mathrm{v}}(G_{S+T} - 1) v_S, \quad \text{where} \quad G_{S+T} = v_{S+T}/v_S.$$

Turning to the balance condition of a pay-as-you-go wage-indexed pension system, one needs distinguish real gross wage u_t from real total labor compensation w_t. To connect the three wages, various tax and contribution rates are introduced. For the time being, we assume that all rates are time-invariant.

Employee's contribution rates; pension: $\tau^{\rm E}$, health: $\theta^{\rm E}$. Employer's contribution rates; pension: $\tau^{\rm F}$: health: $\theta^{\rm F}$. Pension contribution rate $\tau=\tau^{\rm E}+\tau^{\rm F}$. Personal income tax rate: σ.

Though net wages are relevant at calculating replacement ratios, we should also introduce total labor compensation w_t and gross wage u_t. By definition,

$$v_t=(1-\tau^{\rm E}-\theta^{\rm E}-\sigma)u_t \quad \text{and} \quad w_t=(1+\tau^{\rm F}+\theta^{\rm F})u_t,$$

where $\psi=1-\tau^{\rm E}-\theta^{\rm E}-\sigma$ will denote the ratio of net to gross wage: $v_t=\psi u_t$.

Then the system's balance condition is as follows:

$$\tau S u_t = T\beta v_t.$$

We also introduce the *dependency ratio* μ, which is the ratio of the number of pensioners to that of the workers. In our model, $\mu=T/S$ – time-invariant.

Considering a DB system, we have arrived to

Theorem 2. *In a wage-indexed pension system, the balanced pension contribution rate is equal to the product of the dependency ratio (μ) and of the gross accrual rate ($\psi\beta$):*

$$\tau=\mu\psi\beta.$$

Unfortunately, indexing to wages does not prevent the decline of the real value of the benefit, when the average real wage drops: if $v_t<v_{t-1}$, then $b_t<b_{t-1}$. To avoid this problem, between 1975 and 1980 the UK government chose a strange index: the maximum of 1 and the real wage growth coefficient g_t. But this rule overindexed the pensions in progress and was terminated (Barr–Diamond, 2008, Box 5.8, p. 77). In the German point system, a discretionary decision practically excludes this accident. The Swedish NDC system has a built-in balancing mechanism, to maintain sustainability.

A sensible solution to avoid any drop is as follows. The maximum rule is only a conditional plan:

$$b_t^{\rm c}=\max(\beta v_t,\hat{b}_{t-1}).$$

To phase-out excessive benefit rises, the government opens an account, the capital of which is equal to F_t at the end of year t. The government introduces a feedback rule with an appropriately chosen coefficient $\kappa>0$. To compensate for not reducing the benefit in year $t-1$ when $v_{t-1}<v_{t-2}$, the raise in t is correspondingly diminished:

$$\hat{b}_t=\begin{cases} b_t^{\rm c}+\kappa F_{t-1} & \text{if } \hat{b}_{t-1}=\hat{b}_{t-2};\\ b_t^{\rm c} & \text{otherwise.}\end{cases}$$

The account's dynamics is as follows:

$$F_t=F_{t-1}+\tau S u_t - T\hat{b}_t, \qquad F_0=0.$$

To illustrate the operation of our rule, as a starting point, we use the following parameter values as of Hungary, 2016. The dependency ratio is equal to $\mu=20/35=0.571$, the net replacement ratio is equal to $\beta=0.8$. Since $\sigma=0.15$, $\tau^{\rm E}=0.1$ and $\theta^{\rm E}=0.08$, therefore $\psi=0.67$, the pension contribution rate: $\tau=0.571\times 0.67\times 0.8=0.306$.

To highlight the virtue of our proposal, Table 2 displays a real gross wage path with wildly but regularly oscillating growth coefficients: $g_t = 1.02 + (-1)^{t+1}0.04$, i.e. it alternates between 0.98 and 1.06, their geometric average being close to 1.02. With the simple benefit rule (lacking the account), from year 2 to year 3, in terms of the initial gross wage $u_0 = 1$, the benefit drops from 0.565 to 0.554, etc. In the modified system with a feedback coefficient $\kappa = 0.05$, in odd years, the benefit remains the same as previously, but in even years, its value is diminished with respect to the simple rule, e.g. in year 4, $0.576 < 0.587$. The account's capital oscillates with narrow bounds.

Table 2. *Wage-indexed pensions without or with an account*

Year t	Gross wage u_t	Simple benefit b_t	Modified benefit $\hat{b}_t$	Account F_t
1	0.980	0.533	0.533	0.000
2	1.039	0.565	0.565	0.000
3	1.018	0.554	0.565	–0.226
4	1.079	0.587	0.576	0.000
5	1.058	0.575	0.576	–0.009
6	1.121	0.610	0.609	0.000
7	1.099	0.598	0.609	–0.235
8	1.164	0.633	0.622	0.000
9	1.141	0.621	0.622	–0.018

3. Indexing to prices (macro)

In this Section, we investigate the dynamics of pensions when benefits in progress are indexed to prices. Its analysis is more complex than that of indexing to wages, because we have to distinguish the benefits of pensioners retired in different years even after the initial transition is over. Here we already allow for the one-year-lag in valorization. To keep notations simple, now b_t stands for benefit first granted in year t rather than the common value of benefits paid in that year (in Section 2).
Newly awarded benefit with delay:

$$b_t = \beta v_{t-1}, \qquad t = 1, 2, \ldots. \tag{2}$$

Invariant real value of benefit, started in year $t - k$:

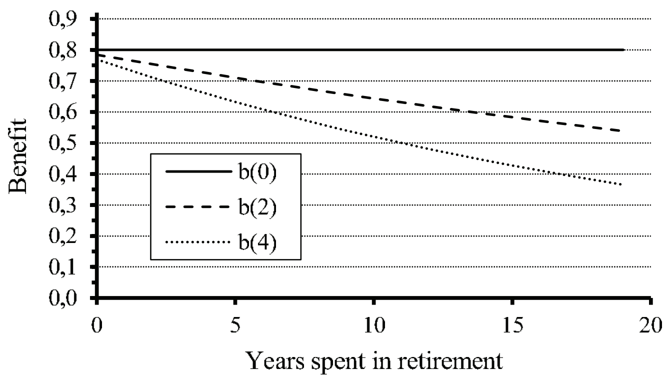
$$b_{t-k} = \beta v_{t-k-1}, \qquad k = 1, 2, \ldots, T-1, \qquad t = 1, 2, \ldots. \tag{3}$$

Again, the predetermined benefits are set as if the system started in $t = -T + 1$:

$$b_0 = \beta v_{-1}, \qquad b_{-1} = \beta v_{-2}, \qquad \ldots, \qquad b_{-T+1} = \beta v_{-T+2}. \tag{4}$$

Figure 1 displays the relative depreciation of old benefits w.r.t. the new ones. For simplicity, here we assume temporarily that the growth rate of the real wage is time-invariant: $v_t = v_{t-1}g$ and compare three benefit paths for real net wage growth rates $100(g-1) = 0, 2, 4$. (The corresponding benefit paths $b(k)$ are indexed by $k = 0, 2, 4$.) The higher the growth rate, the stronger the depreciation: while for zero growth rate, the benefit remains 80% of the real net wage in year of the start; for 4%, the benefit drops to 38% of the foregoing wage at the end.

Figure 1. Relative depreciation



Before determining the new balance conditions, we introduce new concepts, allowing for time-variant growth rates.
Total expenditures in year t:

$$B_t = \sum_{k=0}^{T-1} b_{t-k}.$$

The average benefit and average replacement ratio respectively are equal to

$$\bar{b}_t = \frac{B_t}{T} \quad \text{and} \quad \gamma_t = \frac{\bar{b}_t}{v_t}. \tag{5}$$

The average replacement ratio has a dual role: (i) it measures the average benefit in terms of the net wage and (ii) it transforms the underlying DC to a DB system with time-variant pension contribution rate τ_t. Note that in contrast, if benefits are indexed to wages, then the average benefit ratio is equal to the accrual rate: $\gamma = \beta$.

First we illustrate theoretically and numerically the dependence of the average replacement ratio on the time-invariant wage growth coefficient g. We shall need the concept of *equivalent number of years in retirement*:

$$T_g = \sum_{k=1}^{T} g^{-k} = \frac{1-g^{-T}}{g-1} < T \qquad \text{for} \quad g > 1 \quad \text{and} \quad T_1 = T.$$

Equivalence means that indexing to prices during T years costs the same as indexing to wages during T_g years. With T_g's help, we have

Theorem 3. *For a system where pensions in progress are price-indexed and real wage growth coefficient g is time-invariant, the corresponding time-invariant average replacement ratio is given by the decreasing function*

$$\gamma(g) = \beta \frac{T_g}{T}.$$

A well-known disadvantage of price indexing for the pensioners (which is an advantage for the government) is as follows: the higher the real wage growth rate, the lower the average replacement ratio with respect to a fixed accrual rate. The lag in valorization (2) causes a small part of the drop, and the lagging of pensions in progress behind the initial one in indexation (3) causes the large part of the drop. Quantitatively, with $T = 20$ years spent in retirement, Table 3 demonstrates how the average replacement ratio—in parallel with T_g—drops from $\gamma(0) = \beta = 0.8$ through 0.654 to 0.498 as the growth rate of the real net wages rises from 0 through 2 to 5%. The last column is discussed later.

Table 3.
Average replacement ratio as a function of growth rate of real wages: price indexing

Growth rate of real wages $100(g-1)$	Equivalent number of years in retirement T_g	Net average replacement ratio γ	Pension contribution rate τ
0	20.0	0.800	0.306
1	18.0	0.722	0.276
2	16.4	0.654	0.250
3	14.9	0.595	0.228
4	13.6	0.544	0.208
5	12.5	0.498	0.191

Remark. $\beta = 0.8$.

We turn now to the dynamics of the average replacement ratio when the real wage growth rate is time-variant. Starting with the tautological approach, (5) yields a trivial formula:

$$\frac{\gamma_t}{\gamma_{t-1}} = \frac{\bar{b}_t}{\bar{b}_{t-1} g_t}.$$

In words: the growth coefficient of the average replacement ratio is equal to the ratio of the growth coefficients of the average benefits and of the wages. While the formula holds for any type of indexation; in pure indexing to wages, both sides simplify to 1.

Digging deeper, for pensions indexed to prices, we can express the dynamics of the average benefits and of the average replacement ratio as functions of the underlying wage growth coefficients, respectively. The following recursion is to be used:

$$B_t = B_{t-1} + b_t - b_{t-T}. \tag{6}$$

Hence relying on (4), (5) and (6), the new average replacement ratio is given by

$$\gamma_t = \frac{\bar{b}_t}{v_t} = \frac{\bar{b}_{t-1}}{g_t v_{t-1}} + \beta \frac{v_{t-1} - v_{t-T-1}}{T v_t}. \tag{7}$$

To simplify (7), we use again the cumulated real wage growth coefficient between years $t-T$ and t: $G_t = v_t/v_{t-T}$ which is also equal to the ratio of the next year's youngest and oldest pensions: $G_t = b_{t+1}/b_{t-T+1}$.

Theorem 4. *For time-variant real net wage growth rates and price indexation, the dynamic of average replacement ratio is given by*

$$\gamma_t = \frac{\gamma_{t-1}}{g_t} + \beta \frac{1 - G_{t-1}^{-1}}{g_t T}, \qquad t = 1, 2, \ldots. \tag{8}$$

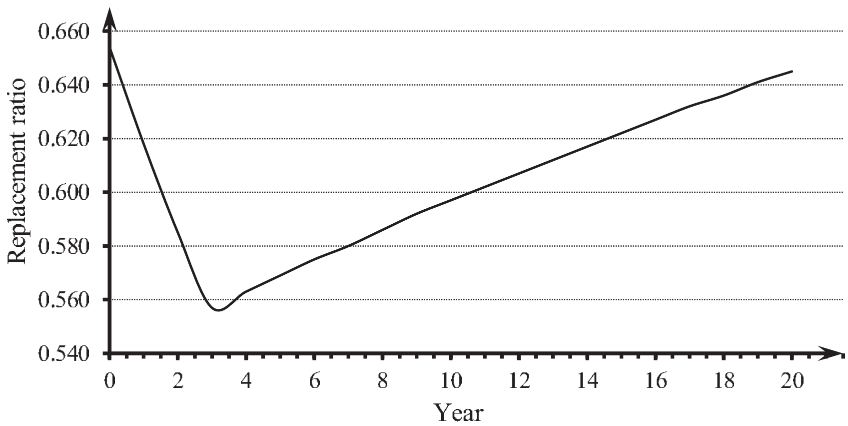
It is worth adding some explanation to (8). The first, dominant term represents past average replacement ratio, scaled-down by the current real wage growth coefficient. The second term represents the impact of the entry of the youngest cohort and of the exit of the oldest cohort, it is typically small, less than $\beta/T = 0.03$ in modulus, which pushes up or down the first term.

Having this formula, we model the impact of an extraordinary real wage hike, similar to that occurring in Hungary during 2016–2018, on the average replacement ratio. We assume that there are two values of the real wage growth coefficients $(1 <) g_{\mathrm{m}} < g_{\mathrm{M}}$, the greater is reached in year $t_0 - 1, t_0, t_0 + 1$:

$$g_t = \begin{cases} g_{\mathrm{m}} & \text{if } t < t_0 - 1 \text{ or } t > t_0 + 1; \\ g_{\mathrm{M}}, & \text{otherwise.} \end{cases}$$

Using the data of Figure 1, and noting that for $g_{\mathrm{m}} = 1.02$ and $g_{\mathrm{M}} = 1.08$, $G_0 = g_{\mathrm{m}}^T$, Figure 2 depicts a stylized process. Starting from a steady state, the real wage explosion reduces the average replacement ratio $\gamma_0 = \gamma(g_{\mathrm{m}}) = 0.654$ to $\gamma_3 = 0.557$ and then γ_t slowly returns to the start.

Figure 2. Replacement dynamics



We shall demonstrate that the temporary drop of the average replacement ratio makes room for a similar temporary reduction of the contribution rate. But as the replacement ratio eventually returns to its former value, so does the contribution rate. To show this, we introduce

Total contributions in year t:

$$I_t = \tau_t S u_t.$$

The new balance condition ($I_t = B_t$) is as follows:

$$\tau_t S u_t = T\bar{b}_t = T\gamma_t v_t = T\gamma_t \psi u_t.$$

We have arrived to

Theorem 5. *The balanced pension contribution rate is given by the product of the dependency ratio (μ) and the average gross replacement ratio ($\psi\gamma_t$):*

$$\tau_t = \mu\psi\gamma_t. \tag{9}$$

Remarks. 1. As is known, (9) always holds, regardless of the form of indexation.

2. Note that following the current Hungarian practice, only the employer's pension contribution rate τ_t^{F} varies in time, therefore ψ is time-invariant. Correspondingly

$$\tau_t^{\mathrm{F}} = \mu\gamma_t\psi - \tau^{\mathrm{E}}, \quad \text{where} \quad \psi = 1 - \tau^{\mathrm{E}} - \theta^{\mathrm{E}} - \sigma. \tag{9'}$$

3. The last column of Table 3 above shows the impact of the time-invariant real wage growth rate on the balanced contribution rate, dropping from 30.6% (for stagnating real wage) to 19.1% (for real wage rising by 5% per year).

Finally, we return to the undiscounted lifetime benefits of the two cohorts introduced in Table 1 but now for the price-indexed ones:

$$C_0^{\mathrm{p}} = \sum_{t=0}^{T-1} b_{S+t}^{\mathrm{p}} = Tb_S^{\mathrm{p}} \quad \text{and} \quad C_1^{\mathrm{p}} = \sum_{t=1}^{T} b_{S+t}^{\mathrm{p}} = Tb_{S+1}^{\mathrm{p}},$$

hence their difference is equal to

$$C_1^{\mathrm{p}} - C_0^{\mathrm{p}} = T\beta(v_S - v_{S-1}) = T\beta(g_S - 1)v_{S-1}.$$

Table 4 presents the differences of lifetime benefits in terms of the initial gross wage, arising for a single-year real wage hike under wage- and price-indexing rules, respectively. The wage hike runs from 0 to 10% in year S, just when the first cohort retired, otherwise real wages grow by 2%. Note that under wage indexing, the difference (given in terms of the initial gross wage $u_0 = 1$) grows moderately, while under price indexing, the unadjusted difference rises rather steeply, showing the lack of robustness.

Table 4. *The differences between subsequent cohorts' lifetime benefits*

Real wage hike $100(g_S - 1)$ %	0	2	4	6	8	10
Wage-indexed diff $C_1^{\mathrm{v}} - C_0^{\mathrm{v}}$	0.762	0.777	0.793	0.808	0.823	0.838
Price-indexed diff $C_1^{\mathrm{p}} - C_0^{\mathrm{p}}$	0	0.627	1.255	1.882	2.510	3.137

4. Indexing to wages (micro)

In Sections 2 and 3, we demonstrated that at macrolevel, the only robust method—meaning that close wage paths yield close benefit paths—is when benefits in progress are indexed to wages. This requires, however, some political courage from the government to (i) reduce the accrual rate β appropriately during rising dependency ratio (neglected in the paper) and (ii) forsake temporary reduction of the contribution rate τ_t during a real wage hike (see above). Moreover, at microlevel, indexing to wages has an unpleasant side effect: since the life expectancies of various income groups widely differ, namely higher earners live longer, therefore the faster the benefits increase, the stronger the income redistribution from the shorter-lived to the longer-lived. We shall show how this can be mitigated by the introduction of *pension progression*.

Working out the necessary changes, for the sake of simplicity, we neglect again the time-variance of real wage growth rates, the problem of transition in indexation and relax the assumptions of homogeneous wages and life expectancies. Let i be the index of a wage group, $i = 1, 2, \dots, n$, $f_i > 0$ be its relative frequency: $\sum_{i=1}^n f_i = 1$ and Q is the age of entry to work. We assume that the real wage of each group grows at the same time-invariant rate as the average gross wage $u_t = g^{t-Q}$, therefore the corresponding type-specific gross wage in year t is equal to the product of a time-invariant constant ω_i (increasing in i) and of the average wage:

$$u_{i,t} = \omega_i u_t, \quad \text{where} \quad u_Q = \sum_{i=1}^n f_i \omega_i = 1. \tag{10}$$

(In Appendix A, we shall cover the general case of type- and time-variant growth coefficients $g_{i,t}$.)

By assumption, everybody retires at age $R = Q + S$, type i lives until $D_i = R + T_i$: ω_i as well as D_i is increasing, and $\sum_{i=1}^n f_i D_i = \bar{D}$.

To eliminate cross-subsidization, one could have type-specific decreasing *average* accrual rates β_i $(i = 1, 2, \dots, n)$ but it would change the order of benefits at the left- and right-hand sides of the bending points. This is avoided by the use of decreasing *marginal* accrual rates, and this diminishes redistribution in the US Social Security (cf. Liebmann, 2002). Following Disney (2004), we approximate progression by the linear combination of proportional and flat benefits. We shall keep average gross and net wages in year t by u_t and v_t, respectively, and denote the share of proportional benefits by α, $0 \le \alpha \le 1$.

The mixed benefits of wage class i are given by

$$b_{i,t} = \beta[\alpha v_{i,t} + (1 - \alpha) v_t], \qquad i = 1, \dots, n. \tag{11}$$

Simplifying the calculations, we retain stationary population. The balance condition is now

$$\tau_\alpha S u_t = \sum_{i=1}^n f_i T_i b_{i,t}.$$

Take the weighted and doubly weighted average times spent in retirement, respectively:

$$\bar{T} = \sum_{i=1}^n f_i T_i \quad \text{and} \quad T^{\mathrm{u}} = \sum_{i=1}^n f_i T_i \omega_i. \tag{12}$$

Obviously, $T^{\mathrm{u}} > \bar{T}$. Substituting (11) and (12) into the balance condition, yields another balance equation:

$$\tau_\alpha S = \psi \sum_{i=1}^{n} f_i T_i \beta[\alpha\omega_i + (1-\alpha)], \quad \text{where} \quad \psi = 1 - \tau^{\mathrm{E}} - \theta^{\mathrm{E}} - \sigma.$$

Thus we have arrived to the generalization of Theorem 2.

Theorem 6. *For pensions in progress indexed to wages and for heterogeneous wage profile (ω_i) and times spent in retirement (T_i), the balanced contribution rate is given by*

$$\tau_\alpha = \frac{\beta\psi[\alpha T^{\mathrm{u}} + (1-\alpha)\bar{T}]}{S}. \tag{13}$$

Remark. As the proportional benefit's share α decreases, so decreases the balanced contribution rate.

We shall now analyze the income redistribution due to heterogeneous earnings and life expectancies. Corresponding to the logic of the pay-as-you-go system, the *type-specific lifetime balance* in year Q should be discounted by the real growth coefficient g, therefore it is defined by

$$z_{i,Q} = \tau_\alpha S\omega_i - \sum_{k=R}^{D_i} g^{-(k-R)} b_{i,k}.$$

Using (11)–(12), the type-specific lifetime balance is given by

$$z_{i,Q} = \alpha(\tau_\alpha S - \beta\psi T_i)\omega_i + (1-\alpha)(\tau S\omega_i - \beta\psi T_i). \tag{14}$$

As an illustration, we consider the traditional homogeneous life expectancies, where $T_i \equiv \bar{T}$, i.e. $T^{\mathrm{u}} = \bar{T}$, i.e. (13) simplifies to $\tau = \psi\beta\mu$, regardless of α. The type-specific lifetime balance is then equal to

$$z_{i,Q} = (1-\alpha)\beta\psi\mu(\omega_i - 1)\bar{T},$$

i.e. those who earn below the average ($\omega_i < 1$), gain ($z_{i,Q} < 0$), the others ($\omega_i \geq 1$) lose ($z_{i,Q} \geq 0$).

Table 5 presents a more realistic but still simple numerical illustration. There are three types differing in their relative gross wages: $\omega_1 = 0.5$; $\omega_2 = 1$ and $\omega_3 = 2.125$; their frequencies are $f_1 = 0.45$, $f_2 = 0.35$ and $f_1 = 0.2$, keeping the average gross wage at unity [(10)]. Let $Q = 25$ and $R = 60$, the corresponding life expectancies be $D_1 = 77$, $D_2 = 80$ and $D_3 = 86.75$ years, resulting in average life expectancy equaling to $\bar{D} = 80$ years. The lower the proportional share, the lower the contribution rate. In addition, we display the type-specific lifetime balances. For a proportional system ($\alpha = 1$), the higher earners and longer-lived are the gainers ($z_{3,Q} < 0$), the others are the losers; this changes with decreasing α to 0.5. (For different parameter values, the picture would be different.)

Table 5. *Progression, contribution rates and type-specific balances*

Proportional share α	Balanced contribution rate τ_α	*Lifetime balance* with life expectancy short $z_{1,Q}$	medium $z_{2,Q}$	long $z_{3,Q}$
1.00	0.340	1.392	1.176	–5.190
0.75	0.331	–0.296	0.882	–0.877
0.50	0.323	–1.984	0.588	3.435
0.25	0.315	–3.672	0.294	7.748
0.00	0.306	–5.360	–0.000	12.060

Up to now we have left out the *fragmentation* of working careers (cf. Augusztinovics and Köllő, 2008) and here we make up this omission. A basic problem of most (but not so much of the US) pension systems is that workers who have long lacunas in their labor histories will receive rather low benefits with respect to those who have no lacunas. We introduce the double-weighted expected contribution length:

$$S^{\mathrm{u}} = \sum_{i=1}^{n} f_i \omega_i S_i.$$

Now (13)–(14) modify respectively into

$$\tau_\alpha = \frac{\beta\psi[\alpha T^{\mathrm{u}} + (1-\alpha)\bar{T}]}{S^{\mathrm{u}}} \tag{13'}$$

and

$$z_{i,Q} = \tau_\alpha S_i \omega_i - \beta[\alpha\psi\omega_i + (1-\alpha)]T_i. \tag{14'}$$

We could illustrate these formulas as well but we forsake it. We also skip the analysis of variable retirement ages.

5. Conclusions

At the end of the main text, we draw some conclusions. We have demonstrated the relative simplicity and fairness of indexing public pensions to wages. Even the adverse impact of temporary drop of real wages on real benefits can be mitigated by an account. We have also demonstrated that the apparently frugal price indexing has a number of pitfalls: In addition to reducing the relative value of old benefits to current wages, it also creates unjustified differences between pension paths of cohorts whose retirement is separated by a real wage shock. This anomaly favors wage indexing over price indexing, but then the government cannot apply the forced reduction of contribution rate during the real wage hike. Furthermore, the government has to weaken the side effect of wage indexing, namely maximizing the perverse redistribution from low-earning short-lived citizen to high-earning long-lived ones. To mitigate this pitfall, the public has to rely on progressive pensions.

Unfortunately, introducing progression often weakens the incentives to report wages. Together with wage indexing, both may undermine the incentives to work longer and strengthen those for early retirement. But the wage indexing is probably close to the social optimum if supplemented with progression. The only remaining problem is: how to phase it in?

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Appendix A. Valorization, indexation and point system

In the main text we avoided wage paths with type-specific real wage growth rates. Now we make up this omission, also define combined wage-price indexing and the point system.

Assume that a worker of type i, born in year 0 enters work at age Q and earns real net wage $v_{i,a}$ at age a, $a = Q, \ldots, R-1$. Her initial pension is given as

$$b_{i,R} = \delta \sum_{a=Q}^{R-1} G_{R-1,a} v_{i,a},$$

where the *valorization multipliers* from year a to year $R-1$ in real terms are equal to

$$G_{R-1,a} = \frac{v_{R-1}}{v_a}, \qquad a = Q, \ldots, R-1,$$

v_a being the economy-wide real wage in year a and δ denoting the *marginal* accrual rate.

At this point we show that—apart from the 1-year lag—(1) is a good approximation of the general formula. Indeed, for our representative worker, who at age a earns the current average net wage v_a, the initial benefit is equal to

$$b_R = \delta \sum_{a=Q}^{R-1} G_{R-1,a} v_a = \delta(R-Q)v_{R-1},$$

thus $\beta = \delta(R-Q)$.

Let ι be a real number between 0 and 1, to be called *wage index weight*. Then raising the real wage growth coefficient to this power, the previous benefit is multiplied by this number to yield the new benefit in progress:

$$b_{i,a} = b_{i,a-1} g_a^{\iota}, \qquad a = R+1, \ldots, D-1.$$

The predetermined benefits are again given.

It is obvious that $\iota = 1, 1/2, 0$ represent wage, wage-price and price indexing rules, being in force in Hungary during the periods 1993–1999, 2000–2009 and 2010–, respectively, as will be described in Table B.1 below.

At this point we turn from the DB to the DC system and fix τ rather than β. In the framework of time-invariant real wage growth rates, we can now formulate Diamond's trade-off between higher indexation and lower initial benefit mentioned at the beginning. Introducing the wage-index-weight-dependent accrual rate $\beta(\iota)$ and the generalized equivalent years spent in retirement

$$T_{g,\iota} = \frac{1 - g^{(\iota-1)T}}{g^{1-\iota} - 1}, \qquad \iota < 1 \quad \text{and} \quad T_{g,1} = T,$$

yields

Theorem A.1. *For a time-invariant real wage growth coefficient g and a given average replacement ratio γ, there is the following trade-off between the wage index weight ι and the accrual rate $\beta(\iota)$:*

$$\beta(\iota) = \gamma \frac{T}{T_{g,\iota}}.$$

Table A.1 displays the trade-off between the wage index weight and the accrual rate under a fixed contribution rate $\tau = 0.25$ for long-run growth coefficient $g = 1.02$. As the wage index weight rises, so decreases the accrual rate from 0.8 to 0.653.

Table A.1. *Trade-of between the wage index weight and the accrual rate*

Wage index weight ι	0	0.25	0.5	0.75	1
Accrual rate $\beta(\iota)$	0.800	0.760	0.723	0.688	0.653

Finally we outline the logic of a point system. In year $t+a$, an i-type worker of age a earns points

$$p_{i,a,t+a} = \frac{v_{i,a,t+a}}{v_{t+a}},$$

i.e. the ratio of her wage to the economy-wide average. Her *cumulated* points earned up to retirement is equal to the sum of these points:

$$\mathbf{p}_{i,R,t+R} = \sum_{a=Q}^{R-1} p_{i,a,t+a}.$$

The value of one point in year $t+a$, x_{t+a} yields a benefit

$$b_{i,a,t+a} = \mathbf{p}_{i,t+a,R} x_{t+a}, \qquad a = R, \ldots, D_i - 1.$$

Note that in the point system, there is neither wage indexing nor price indexing nor their mixture; for example, denoting the cross-sectional profile in year t by $(b_{i,a,t})_a$, the point value x_t is determined from a complex balance condition:

$$\tau S w_t = \sum_{i=1}^{n} f_i \sum_{a=R}^{D_i} b_{i,a,t} = x_t \sum_{i=1}^{n} f_i T_i \mathbf{p}_{i,R,t}.$$

Appendix B. Selected statistics

This Appendix contains two sets of time series of Hungary and of several EU countries, respectively.

Table B.1 displays the historical time series of the Hungarian developments during 1993–2018: the more so because the GDP and the net wage growth rates were very turbulent, and indexation rules to wages, to wages and prices and to prices followed each other, yielding oscillating net average replacement ratios. We inform the reader on other factors in the last column. Just concentrating on the latest development, from 2015 to 2018, real net average wage grew by an astonishing 28% while the GDP only grew by 10%, pensions (mostly price-indexed pensions in progress) only by 6%.

Table B.1. *Output, real wage and real pension dynamics: Hungary: 1993–2018*

Year	Real growth rate of GDP	net wage	pension	Net replacement rate	Comments
t	$100(g^{\mathrm{y}}-1)$	$100(g^{\mathrm{v}}-1)$	$100(g^{\mathrm{b}}-1)$	γ_t	
indexation to wages					
1993	–0.8	–3.9	–4.6	0.603	
1994	3.1	7.2	4.7	0.594	E: change in PIT
1995	1.5	–12.2	–10.1	0.619	change in delay
1996	0.0	–5.0	–7.9	0.593	
1997	3.3	4.9	0.4	0.563	
1998	4.2	3.6	6.2	0.578	E
1999	3.1	2.5	2.1	0.592	
Swiss indexation (half wage+half price)					
2000	4.2	1.5	2.6	0.591	
2001	3.8	6.4	6.6	0.591	+ raise
2002	4.5	13.6	9.8	0.573	E++ raise
2003	3.8	9.2	8.5	0.568	+ 1 week pension
2004	4.9	–1.1	3.9	0.600	+ 2 weeks pension
2005	4.4	6.3	7.9	0.611	+ 3 weeks pension
2006	3.8	3.6	4.5	0.623	E + 4 weeks pension
2007	0.4	–4.6	–0.3	0.668	
2008	0.8	0.8	3.4	0.691	
2009	–6.6	–2.3	–5.7	0.672	no 13th month benefit
indexation to prices					
2010	0.7	1.8	–0.9	0.651	E
2011	1.8	2.4	1.2	0.647	
2012	–1.7	–3.4	0.1	0.670	
2013	1.9	3.1	4.5	0.676	overindexation**
2014	3.7	3.2	3.2	0.676	E+ overindexation
2015	2.9	4.3	3.5	0.668	overindexation
2016	2.1	7.4	1.4	0.637	start of wage explosion
2017	4.1	10.2	3.0	0.583	wage explosion continued
2018*	4.8	8.0	2.0	0.551	wage explosion ends?

Source: ONYF (2016, Table 1.3, p. 16), new data are added, *: forecast, E = election, **: when the inflationary forecast was higher than the actual, the additional benefit rise was not deducted.

Table B.2 displays selected statistics on real net wage growth in five EU countries between 2001 and 2016, for one-earners without children. There is an ongoing debate whether the official Hungarian data (also used in valorization) are consistent or not to the EU statistics. Column 2 of Table B.2 does not conform to column 3 of Table B.1, either but their qualitative behavior are the same. Germany and Czechia stand out with smooth wage growth. Latvia and Romania had wage growth data even more exotic than the Hungarian. It would be interesting to know how the pensions in progress in Latvia and Romania reacted to them.

Table B.2. *Growth of real net earnings in selected EU-countries,%*

Year	Hungary	Germany	Latvia	Czechia	Romania
2001	1.9	*	2.8	3.5	9.7
2002	10.8	1.0	4.3	5.3	–3.0
2003	10.1	–0.2	6.5	4.5	9.4
2004	–0.2	2.5	5.4	3.3	7.6
2005	4.7	–0.2	9.7	2.7	10.1
2006	2.6	–1.1	15.2	6.7	6.6
2007	–4.9	–2.1	18.3	3.7	15.5
2008	3.1	1.1	6.8	1.3	16.4
2009	1.1	0.3	–2.0	4.6	–1.5
2010	8.6	5.5	–2.6	0.5	1.4
2011	–4.1	0.1	–0.6	–0.3	3.9
2012	1.5	0.0	2.0	–0.8	–3.8
2013	2.6	–0.2	4.5	–1.6	4.2
2014	4.0	1.0	*	2.2	5.3
2015	3.7	1.5	7.5	2.2	11.5
2016	5.6	*	*	2.5	*

Remark. Single person without children