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When and How to Subsidize Tax-Favored Retirement Accounts?

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When and How to Subsidize Tax-Favored Retirement Accounts?*

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Abstract

When and how to subsidize tax-favored pension accounts? To defend myopic workers against themselves, the government introduces a mandatory system but to help savers, it adds tax-favored retirement accounts. If the mandatory system is progressive, then a proportional voluntary system can beneficially dampen the redistribution. If the mandatory system is proportional, then a progressive voluntary system may raise the old-age consumption of the lower-paid. But if both the mandatory and the voluntary systems are proportional and the ceiling is high (as is the case in Hungary), then the latter does not diminish the tension of the mandatory system.

Keywords: mandatory pensions, tax-favored retirement accounts, voluntary contributions, subsidies.

JEL: H55, D91

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Mikor és hogyan támogassuk az önkéntes nyugdíjrendszert?

ANDRÁS SIMONOVITS

Összefoglaló

Mikor és hogyan támogassuk az önkéntes nyugdíj-tagdíjakat? A rövidlátó dolgozók értelmében a kormányzat kötelező nyugdíjrendszert vezet be, de a takarékos dolgozók érdekében ezt kiegészíti önkéntes nyugdíjszámlákkal. Ha a kötelező rendszer degresszív, akkor egy arányos önkéntes rendszer jótékonyan csökkentheti az újraelosztást. Ha a kötelező rendszer arányos, akkor egy degresszív önkéntes rendszer emelheti a kiskeresetű dolgozók időskori fogyasztását. De ha mind a kötelező, mind az önkéntes rendszer arányos, és a korlátok magasak (mint ma Magyarországon), akkor az önkéntes rendszer nem csökkenti a kötelező rendszer feszültségeit.

Tárgyszavak: kötelező nyugdíj, önkéntes nyugdíj, tagdíj, támogatás, adókedvezmény

1. Introduction

In most developed countries, in addition to the mandatory (funded and/or unfunded, public or private) pension system, a voluntary pension system exists, providing tax and contribution subsidies. The voluntary pension system is formed by *tax-favored retirement accounts*. In the default case, these subsidized savings cannot be withdrawn until the owner retires. The proponents of such systems justify these subsidies like this: a mandatory system does not and cannot ensure high enough pensions, and the mostly myopic workers must be made interested in raising their old-age incomes through a voluntary system. The opponents are afraid that these subsidies are poorly targeted, mostly subsidize the well-paid savers, while worsening the burden of the others by increasing the *tax expenditures*. Up to now these tax expenditures have generally been quite low, thus they may be neglected, but in a possible contraction of the mandatory system they may become much higher. In this paper, I will discuss the issue in a very simple model. Since there are no other taxes in the model, I will write earmarked taxes rather than tax expenditures, pretending that a special tax finances the subsidies. My results are as follows: as a complement to a progressive mandatory system, (where the benefit-mandatory contribution ratio is decreasing) the voluntary system is acceptable, because the transfers in the two systems counterbalance each other. However, added to a generous proportional (contributive) mandatory system, the subsidization of a voluntary system is debatable, therefore it is worth limiting the voluntary contributions and making the system progressive.

In the paper we assume that the personal income tax and the pension systems are socially accepted, the citizens pay their due taxes and mandatory contributions, and both systems are balanced. This assumption hardly applies to certain countries including Hungary, and this weakens the force of the paper's statements. For example, one can argue that the Hungarian personal income tax system is so progressive and tax and mandatory contribution evasion is so widespread that the transfers received in the voluntary system are only modest compensations to those who shoulder the whole system. To get a balanced answer, a model richer than the present one is needed.

In the remaining part of the Introduction, I will first survey the Hungarian literature on the voluntary pension system, then outline my model, and finally give an overview of the English-language literature.

In Hungary, tax-favored retirement accounts were set up at the end of 1993, when the monopillar mandatory system was very progressive and the personal investment possibilities were very limited. On the other hand, at the beginning, the ceiling on the voluntary contribution was higher than even the average gross wage, and half of the voluntary contribution could be regained through the personal income tax, generating excessive subsidization. To tackle this problem, first, the value of ceiling was inflated away, then in 1998 a radical nominal reduction was introduced. In January 2000, the ceiling was redefined (now about 21% of the annual average gross wage), and since January 2006 the subsidy has been transferred to the member's tax-favored account rather than returned directly to his bank account. Since 2000, the return rate (later the matching rate) has been reduced from 50 to 30%. To have a better understanding, it is worth describing the new system in terms of the old one: adding the subsidy to the voluntary contribution, one can define an enlarged voluntary contribution, from which the subsidy is returned to the member. The employers can also pay a voluntary

contribution (up to the minimum wage until 2007 and half of it since then) which is exempt from the employer's social security contribution. (In 2008, the Hungarian employer's total social security contribution rate, including health contribution, was 32.5%.)

Notwithstanding the huge per-capita subsidies, only about 35% of the workers have a tax-favored retirement account. The intensity of the participation is even lower: only 1.2 and 2.4% of the nationwide average gross wage are paid as voluntary contributions by the employee and the employer, respectively, much below the ceilings. At the same time, the subsidy amounts to about 0.1% of the GDP, a number never mentioned in Hungarian sources.

Several readers of the earlier versions of the study called my attention to the fact that recently a new subsidy was added: unlike traditional savings (including repaying mortgages), the voluntary contributions are exempt from tax on interest. The latter tax rate is now 20%, the impact of which is accumulated during the life time. Even calculating with a modest nominal interest rate of 5%, the tax exemption amounts to 1% of the total assets (equaling to 3% of the GDP), thus raising the total relief by 30%.

Very few papers have studied the tax-favored retirement accounts of the countries in transition, especially from a theoretical point of view. We mention four Hungarian empirical studies: Gál (1998), Vidor (2005), Ágoston and Kovács (2007) and Matits (2008). Among others, Gál (1998, p. 29) emphasized that (i) the members of the voluntary funds are much better paid and more educated than the national average; (ii) the participation rate of the self-employed in voluntary pension funds is rather low, therefore voluntary benefits will not make up the missing mandatory benefits for the self-employed. Vidor (2005) estimated the behavior equations of the Hungarian voluntary system. She found that "in Hungary, savings paid to the tax-favored accounts have not crowded out other savings." Using queries, Ágoston and Kovács (2007) demonstrated the very limited role self-reliance plays in Hungary. Finally, Matits (2008) argued for a wide participation in the tax-favored retirement system, emphasizing the low level of mandatory pensions for the lower paid and the low rate of replacement for the very well-paid. The bulk of the experts have defended or still defend the present system, emphasizing the virtues of self-reliance and underplaying the inequity of such a system. Some of the former proponents, however, have recognized the foregoing flaws, and have become opponents.

We construct the simplest possible model and try to evaluate the current Hungarian system and others. We assume that the workers differ in their earnings and discount factors, but we neglect the positive correlation (causal relation) between earnings and life expectancy or between replacement rate and the relative length of the retirement period. Maximizing their subjective utility functions, the workers determine their voluntary contributions (up to the ceiling) and of their traditional old-age saving. On the other hand, the government determines the values of the parameters to maximize the social welfare function, which is the mean value of the objective (undiscounted) utility functions.

As a technical simplification, we consider a mature system, where the mandatory and voluntary contributions on the one hand and the benefits and matching on the other hand, have been proceeding since decades, according to stable proportions. To simplify calculations, we neglect growth and interest. According to Hungarian data,

real yields were not much higher than the growth rate of the real wages, therefore such simplification is acceptable. We underline the obvious but often neglected fact that the subsidies and the tax exemptions are mainly financed from the workers' taxes.

Recently, in Hungary, from a strongly progressive one, the newly awarded mandatory pension benefit has become closely proportional to the individual mandatory contribution, while the tax-favored employee's or the contribution-exempt employer's subsidy has remained proportional to the corresponding voluntary contributions, both up to quite high limits. The voluntary system is mostly used by the well-paid, taking advantage the generous subsidies. Repeating my proposal: in order to eliminate this perverse redistribution, the tax-favored system should be transformed into a progressive one, with raised initial and reduced continued matching rates and ceiling on voluntary contributions, maintaining or even raising the earmarked tax rate, especially if the mandatory contribution rate is reduced.

We have used a similar methodology in a work-in-progress paper (Simonovits–Tóth, 2009), where the interactions among reporting earnings and the degree of redistribution in the personal income tax and mandatory pension system are investigated. We have also distinguished between subjective and objective utility functions, studied the impact of the discount, but have not separated the tax-favored and other retirement savings. (At the same time, here the tax system is narrowed down to the earmarked tax, financing subsidies.)

Concerning mature market economics, a lot of papers have studied the issue. In most Anglo-saxon countries, where the mandatory public pension system (called Social Security in the US) is modest, the better-paid half of the workers participate in various semi-mandatory—semi-voluntary pension systems. In addition to the latter, there are genuinely tax-favored retirement accounts, called Individual Retirement Accounts (IRA) and 401(k).

There is another difference: in Hungary, workers pay their voluntary contributions from after-tax earnings, therefore at the withdrawal from these accounts after retirement, no tax needs to be paid. On the other hand, in the Anglo-saxon world, workers pay their voluntary contributions from pre-tax earnings and they must pay personal income taxes at withdrawal.

Among the large number of US studies, we single out the following ones: Poterba et al. (1996) estimate that the introduction of tax-favored retirement accounts significantly increase total savings, while Engen et al. (1996) find the opposite. Trying at a synthesis, Hubbard and Skinner (1996) guess that both trends are present but the positive trend outweighs the negative. Bernheim (1999) gave an excellent survey on the topic. Love (2007) analyzed the impact of the age, the matching rate, the vesting policies and the withdrawal penalties on the participation rate.

Börsch-Supan et al. (2008) studied the reform of the German system, which is similar to the Hungarian system in many respects. Within the Riester-reform, the replacement rate of the public pension (i.e. the ratio of pensions to net earnings) is moderated, and the resulting gap is to be financed by the voluntary pillar. The reform has been phased in, and by 2008, every worker may contribute maximum 4% of his gross wage and receive various benefits (Table 1, p. 298). According to the paper, the reform proved to be a success, though the share of the low-paid has been lagging behind the high-paid: 7.3 vs. 20.9%, both being quite low (Table 5, p. 310.)

Modeling the much more complex British system, Sefton et al. (2008) ask a similar question: what is the impact of the introduction of pension credit on other pension savings? According to their model, there was only a small increase, because the increase in the savings of the lower paid induced by the pension credit was almost counterbalanced by the decrease in the savings of the higher paid. OECD (2005) provides a useful overview.

Even more complex models are used by Imrohoroglu et al. (1998) and Fehr et al. (2008). The latter emphasizes the uncertainty of earning paths and longevity, and quantifies the reduced quality of insurance following the setting up voluntary pension system. Admitting the virtues of these complex models, we still hope that our toy model has its own advantage of being transparent.

The excessive discount of future benefits have already been taken into account and ‘corrected’ by Feldstein (1987) in his welfare comparison of the flat and means-tested pensions. (In a flat pension system everybody receives the same benefit, regardless any other pension, while means-tested pension only tops up one’s other pensions to a modest level.) Cremer et al. (2008) lie very close to our approach, also emphasizing the difference between subjective and objective utility functions as well as the credit constrained life-cycle saving, but in their paper the main role is played by the labor supply rather than by the voluntary contribution to the tax-favored retirement account.

Our approach is orthodox, because it heavily relies on time-consistency: as there is no new information, the workers do not change their saving behavior. Less orthodox models (e.g. Laibson, 1998; Diamond and Kőszegi, 2003) employ the hyperbolic discounting when explaining and evaluating the voluntary pension system. To give a simple example: somebody plans to pay monthly voluntary contributions of 10 units during 480 months to get additional benefit of 20 units during 240 months. But he immediately realizes that if he skips the first month voluntary contribution, then he only loses 0.046 units/month, therefore he may safely skip it. But what happens if he goes on in the second, third etc. month?

Using behavioral economics, Choi et al. (2004) also find a quite unorthodox behavior: if the default option is changed, and the new employees are automatically enrolled into a pension fund, from which they can opt-out, then a much higher share will stay in the voluntary system than in the original default. Being partial equilibrium models, the latter models neglect the tax burden of such schemes.

The structure of the remainder of the present paper is as follows: 2. The model framework. 3. Simple cases. 4. Numerical examples. 5. Conclusions.

2. The model framework

In this Section, we outline the model framework. First we determine the optimal voluntary contributions and savings chosen by the individual workers, then we determine parameter values of the welfare maximizing mandatory and voluntary systems.

Maximizing individual utility

We shall make the following extreme, nevertheless meaningful assumptions. The population and the economy are stationary, traditional saving does not yield interest. Every

young-aged individual works and every old-aged individual is retired. Every worker is employed for a unit time period and every pensioner enjoys his retirement for a period of length μ , $0 < \mu < 1$. (In practice, the more one earns, the longer he lives on average; and the retirement age depends on the pension system, but here we neglect these relations.) Most existing systems superfluously differentiate between employer's and employee's mandatory contributions, but we assume a unified mandatory contribution. Contrary to practice, we prefer the total wage cost w to gross wages (their difference is the employer's contribution) and we calculate on its basis. Thus we assume that a worker with wage w pays a positive mandatory contribution τw , at last up to a ceiling $w_x > 0$. In addition, the worker with wage w pays an *earmarked tax* θw into the budget, financing the voluntary pensions.

In addition to his wage, the worker has another parameter called *discount factor*: δ . We assume that some type (w, δ) prefers additional benefits over the mandatory ones, therefore he pays a *voluntary contribution* r over the mandatory contribution, where $r \in [0, r_x]$, and $r_x \geq 0$ is the *ceiling* on voluntary contribution. The government matches the voluntary contribution r according to a *matching-voluntary contribution* function $a(r)$. As we mentioned in the Introduction, that system is equivalent to the previous one, where part of the voluntary contribution was returned directly to the worker. Indeed, if the government immediately returns a from the extended voluntary contribution r , then this is equivalent to another system where the voluntary contribution is only $r - a$ but the government adds a to the account.

The pension paid as a life annuity consists of two terms: the earnings-related mandatory benefit $b(w)$ and the voluntary pension $[r + a(r)]/\mu$. (As a matter of fact, voluntary pensions are seldom paid as life annuity, but this is irrelevant, because we do not discuss the distribution of consumption within the retirement period.)

Finally, there are types for whom even the maximal voluntary contribution r_x called ceiling and the corresponding maximal subsidy a_x are insufficient. These types can *traditionally save* an additional $s \geq 0$. We assume that the efficiency of this traditional saving is the same as that of the mandatory system, i.e. the corresponding life annuity is s/μ . Note that for an optimizing individual, $s > 0$ implies $r = r_x$!

The instantaneous consumption of a worker and of a pensioner are, respectively

$$c = w - \tau w - \theta w - r - s \quad \text{and} \quad d = b(w) + [r + a(r) + s]/\mu.$$

(Both c and d are positive. Of course, the instantaneous old-age consumption d means a lifetime pensioner consumption μd .)

We turn to the individual optimization. The *subjective* lifetime utility function of a type (w, δ) consists of two terms: (i) the utility $u(\cdot)$ of instantaneous worker consumption c and (ii) the utility $\mu\delta u(d)$ of the pensioner's instantaneous consumption d . Here δ is the discount factor, $0 < \delta < 1$. In sum:

$$\hat{Z}(w, \delta, c, d) = u(c) + \mu\delta u(d).$$

The individual determines the pair (voluntary contribution, saving) $[r(w, \delta), s(w, \delta)]$ by maximizing his lifetime utility $\hat{Z}(w, \delta, c, d)$ under the lifetime budget constraint. Partly for the sake of simplicity, partly for bounded rationality, we assume that each worker takes the earmarked tax rate as given, i.e. does not consider the impact of his

or others' choices. Substituting the consumption equations into \hat{Z} , provides subjective utility in another form:

$$Z(w, \delta, r, s) = u(w - \tau w - \theta w - r - s) + \mu \delta u(b(w) + [r + a(r) + s]/\mu).$$

The worker determines his *optimal* voluntary contribution \tilde{r} and saving \tilde{s} by taking the partial derivatives with respect to decisions r and s . (To avoid lengthy notations, we shall rarely use tilde for the optimum.) We must take into account the possibility of corner solutions. We assume that $b(w)$ and $a(r)$ are increasing concave functions, at least in the intervals $w_m \leq w \leq w_x$ and $0 \leq r \leq r_x$, respectively, where w_m is the minimal wage. Moreover, $b(0) \geq 0$ and $a(0) = 0$. To minimize the number of cases, for the time being, we assume that $b(w)$ and $a(r)$ are smooth functions. Here are the cases to be distinguished:

Zero voluntary contribution, zero saving, $r = 0, s = 0$:

$$Z'_r(w, \delta, 0, 0) = -u'(c) + \delta u'(d)[1 + a'(0)] \leq 0.$$

Positive voluntary contribution below ceiling, zero saving, $0 < r < r_x, s = 0$:

$$Z'_r(w, \delta, r, 0) = -u'(c) + \delta u'(d)[1 + a'(r)] = 0.$$

Maximal voluntary contribution, zero saving, $r = r_x, s = 0$:

$$Z'_s(w, \delta, r_x, s) = -u'(c) + \delta u'(d) \leq 0.$$

Maximal voluntary contribution, positive saving, $r = r_x, s > 0$:

$$Z'_s(w, \delta, r_x, s) = -u'(c) + \delta u'(d) = 0.$$

Macro framework

In our model, workers have two characteristics: w and δ . We assume that their joint probability distribution is given by $(f_i)_{i=1}^I$ (possibly $i = (j, k)$) on the grid-points of the rectangle $w_m \leq w \leq w_x$ and $\delta_m \leq \delta \leq \delta_x$.

We assume that the mandatory contribution covers the mandatory pension expenditure, while the earmarked tax finances the subsidies. In formula:

Balance of the mandatory pensions

$$\sum_{i=1}^I f_i [\tau w_i - \mu b(w_i)] = 0.$$

Balance of the earmarked taxes

$$\sum_{i=1}^I f_i [\theta w_i - a(r(w_i, \delta_i))] = 0.$$

Maximization of the social welfare function

We also assume that the country is managed by a benevolent government which sets its parameters so as to maximize an appropriately defined social welfare function. First of all, it removes discounting, and replaces subjective with *objective* utility functions:

$$U(w_i, \delta_i, c_i, d_i) = u(c_i) + \mu u(d_i).$$

(Note that U is independent of δ_i but to signal the second characteristic of the individual, we still keep δ_i .)

The utilitarian social welfare function is the average of the individual objective utility functions, taken at the optima:

$$V = \sum_{i=1}^I f_i U(w_i, \delta_i, \tilde{c}_i, \tilde{d}_i).$$

If the government has a more egalitarian preference, it can choose a strictly concave scalar–scalar function ψ , and rely on a generalized utilitarian social welfare function (cf. Fehr et al., 2008):

$$V = \sum_{i=1}^I f_i \psi(U(w_i, \delta_i, \tilde{c}_i, \tilde{d}_i)).$$

The government looks for a mandatory contribution rate τ , an earmarked tax rate θ , and a pair of benefit and matching functions $b(\cdot), a(\cdot)$, which maximize the social welfare function under the budget constraints.

In the continuation, it is useful to apply a simple utility function, namely CRRA: $u(c) = \sigma^{-1} c^\sigma$, where $\sigma < 0$. As a special limiting case ($\sigma = 0$), Cobb–Douglas: $u(c) = \log c$ can also be very useful.

3. Simple cases

In this Section we shall work with homogeneous or inhomogeneous linear benefit and matching functions. We shall start with the homogeneous case.

Bounded homogeneous linear benefit–wage–function

$$b(w) = \beta \min(w, w_x),$$

where $\beta > 0$ is the gross replacement ratio. Such is the case with the proportional systems, functioning, for example, in Sweden and planned to be introduced in Hungary from 2013.

Bounded homogeneous linear matching–voluntary contribution function

$$a(r) = \alpha \min(r, r_x),$$

where r_x is the voluntary contribution’s ceiling, $a_x = \alpha r_x$ is the subsidy’s ceiling. Then $a(r) = \min(\alpha r, a_x)$. The Hungarian and the US voluntary systems are good examples.

Preliminaries

Since $u'(c) = c^{\sigma-1}$, therefore for the optimal consumption pair with matching,

$$c^{\sigma-1} = \delta(1 + \alpha)d^{\sigma-1}, \quad \text{i.e.} \quad d = [\delta(1 + \alpha)]^{1/(1-\sigma)}c.$$

We shall need the ratio of the optimal old- and young-age consumption:

$$\gamma(\delta, \alpha) = [\delta(1 + \alpha)]^{1/(1-\sigma)},$$

With this notation, the optimum condition reduces to

$$d = \gamma(\delta, \alpha)c.$$

For the homogeneous linear case, the balance equations are also simple: for example, $\mu\beta = \tau$.

Inserting the consumption functions into the optimality conditions, after rearrangement, for any given θ , we obtain an optimum for each case. Four cases are to be distinguished.

Zero voluntary contribution, zero saving

Inserting equations $d = \beta w$ and $c = (1 - \tau - \theta)w$ into inequality $d > \gamma(\delta, \alpha)c$, yields

$$d = \beta w > \gamma(\delta, \alpha)(1 - \tau - \theta)w$$

determining domain 1 in the (w, δ) -plane, regardless of the wage.

Positive voluntary contribution, zero saving

Inserting equations $d = \beta w + (1 + \alpha)r/\mu$ and $c = (1 - \tau - \theta)w - r$ into $d = \gamma(\delta, \alpha)c$, yields the optimal voluntary contribution:

$$r = \frac{\gamma(\delta, \alpha)(1 - \tau - \theta) - \beta}{\gamma(\delta, \alpha) + \mu^{-1}(1 + \alpha)}w,$$

assuming $0 \leq r \leq r_x$, defining domain 2, depending on the wage.

Maximal voluntary contribution, zero saving

Inserting the equations into the inequality yields $\gamma(\delta, 0)c \leq d < \gamma(\delta, \alpha)c$, i.e.

$$\frac{\gamma(\delta, 0)(1 - \tau - \theta) - \beta}{\gamma(\delta, 0) + \mu^{-1}(1 + \alpha)}w \leq r_x < \frac{\gamma(\delta, \alpha)(1 - \tau - \theta) - \beta}{\gamma(\delta, \alpha) + \mu^{-1}(1 + \alpha)}w,$$

defining domain 3.

Maximal voluntary contribution, positive saving

Inserting equations $d = \beta w + [(1 + \alpha)r_x + s]/\mu$ and $c = (1 - \tau - \theta)w - r_x - s$ into equation $d = \gamma(\delta, 0)c$, yields the optimal saving:

$$s = \frac{\gamma(\delta, 0)(1 - \tau - \theta)w - \beta w - [\gamma(\delta, 0) + \mu^{-1}(1 + \alpha)]r_x}{\gamma(\delta, 0) + \mu^{-1}}.$$

We must require $s \geq 0$, otherwise the worker would pay his voluntary contribution from credit. (Indeed, this anomaly occurred *en mass* in the US, where too many workers

financed their voluntary contributions from mortgages, until the recent collapse of the credit market.) We have obtained domain 4.

Before returning to the tax balance, we must take into account that our *conditional* voluntary contributions depend on the tax rate, in formula: $r_i(\theta)$, $i = 1, \dots, I$. Thus we have obtained an implicit equation for the balanced tax rate θ° :

$$\sum_{i=1}^I f_i[\theta w_i - \alpha r_i(\theta)] = 0,$$

the solution of which demands further analysis.

Introductory examples

To help the understanding, let us begin with the simplest cases. We begin our analysis with the *traditional life-cycle saving*, when there is not even a mandatory pension. Then

$$\hat{c} = \frac{w}{1 + \mu\gamma(\delta, 0)}, \quad \hat{d} = \frac{\gamma(\delta, 0)w}{1 + \mu\gamma(\delta, 0)} \quad \text{and} \quad \hat{s} = \frac{\gamma(\delta, 0)w}{1 + \mu\gamma(\delta, 0)}.$$

Let us continue with the *first-best solution*. If it were possible to achieve it, then the optimal pair and the mandatory contribution rate corresponding to $\delta^* = 1$ would be

$$c^* = d^* = \frac{w}{1 + \mu}, \quad \tau^* = \frac{\mu}{1 + \mu},$$

when traditional saving is zero: $s^* = 0$. It is remarkable that the worker consumption is lower and the pensioner consumption is higher for the first best than in the pension-free optimum: $c^* < \hat{c}$ and $d^* > \hat{d}$.

The introduction of the first-best, however, would be strongly opposed by the myopes (there would be excessive avoidance of mandatory contributions, use of disability and early retirement etc.), therefore the government chooses a lower discount factor $\delta^\circ < 1$ and a corresponding mandatory contribution rate

$$\tau = \frac{\mu\gamma(\delta^\circ, 0)}{1 + \mu\gamma(\delta^\circ, 0)}$$

(dropping $^\circ$ here). The type (w, δ) will then choose the subjectively optimal consumption pair and saving

$$c^\circ = \frac{w}{1 + \mu\gamma(\delta^\circ, 0)} > c^* \quad d^\circ = \frac{\gamma(\delta^\circ, 0)w}{1 + \mu\gamma(\delta^\circ, 0)} < d^*, \quad s^\circ = \frac{\mu[\gamma(\delta, 0) - \gamma(\delta^\circ, 0)]_+ w}{(1 + \mu\gamma(\delta^\circ, 0))(1 + \mu\gamma(\delta, 0))}$$

where x_+ is the positive part of the real number x : $x_+ = x$ if $x \geq 0$, 0 otherwise. (For discount factors lower than the government's, there would be no saving at all.) That would be, however, too low to the government and it introduces an earmarked tax rate θ , which covers the resulting subsidies: $\theta = \alpha\bar{r}$, where average wage is taken as unity. The government's hope is that at least some type will increase its total saving (i.e. the voluntary contribution plus the traditional saving).

We shall discuss very briefly the case when *every worker has the same discount factor*, and it is lower than the government's compromise: $0 < \delta < \delta^\circ < 1$. We also assume that the matching rate α is high enough that everybody uses it: $(1 + \alpha)\delta > \delta^\circ$, and we drop the voluntary contribution's ceiling.

In our case, the voluntary contributions are proportional to the wages, $r = \rho w$, therefore the earmarked tax rate is simply $\theta = \alpha\rho$. The pair of consumption are given by

$$c = [1 - \tau - (1 + \alpha)\rho]w \quad \text{and} \quad d = [\tau + (1 + \alpha)\rho]w/\mu.$$

Substituting into the optimality conditions:

$$[\tau + (1 + \alpha)\rho]\mu^{-1} = \gamma(\delta, \alpha)[1 - \tau - (1 + \alpha)\rho],$$

hence

$$\rho = \frac{\gamma(\delta, \alpha)(1 - \tau) - \tau\mu^{-1}}{(1 + \alpha)[\mu^{-1} + \gamma(\delta, \alpha)]}.$$

Inversely, it is easy to verify that such a system is equivalent to another, where the mandatory contribution rate τ is raised to $\tau' = \tau + \rho(1 + \alpha)$ and the tax-favored system is closed down: $\rho' = 0$.

If the workers can as easily nudged as the literature on bounded rationality claims, then such a simple trick with the opposite direction can make the pension system popular. Or does this trick only work in the small?

From now on we shall consider more complex cases, where the individual discount factors differ but the cases are simple enough to yield analytical treatment.

Proportional mandatory pensions–proportional subsidies

We shall start the in-depth analysis with proportional mandatory pensions–proportional subsidies. Now we have only two types, L and H with relative frequencies f_L and f_H , wages w_L and w_H , and pensions $b_L = \beta w_L$ and $b_H = \beta w_H$ and with increasing discount factors: $0 < \delta_L < \delta_H < 1$. We shall call the types *myope* (L) and *saver* (H). Since the myopes' earning is less than equal to the savers', we assume $w_L \leq w_H$. As a normalization, we assume that the average wage is unity: $f_L w_L + f_H w_H = 1$. We also assume that the government chooses its discount factor between the two types': $\delta_L < \delta^\circ < \delta_H$, but it sets up tax-favored pension funds with a matching rate $\alpha \geq 0$, and ceiling r_x on the voluntary contributions.

Asymmetric system

To simplify the calculations, first we also assume that the matching rate is so low that the myopes do not participate at the voluntary pensions: $\delta_L(1 + \alpha) \leq \delta^\circ$: *asymmetric system*. For the time being, let us assume that the ceiling is so high that the savers' voluntary contribution is lower than the ceiling: $0 < r_H < r_x$, i.e. $s_H = 0$. Moreover, the optimality condition holds for H: $d_H = \gamma(\delta_H, \alpha)c_H$. There is another constraint: the savers do *not* pay such a high voluntary contribution implying that their young-age consumption is lower than their old-age consumption: $d_H \leq c_H$, i.e. $\delta_H(1 + \alpha) \leq 1$. Then the earmarked tax balance is very simple: $\theta = f_H \alpha r_H$. Therefore $c_H = (1 - \tau)w_H - (1 + \alpha f_H w_H)r_H$ and $d_H = b_H + (1 + \alpha)r_H/\mu$. Substituting c_H and d_H into H's optimum condition:

$$\beta w_H + (1 + \alpha)r_H/\mu = \gamma(\delta_H, \alpha)[(1 - \tau)w_H - (1 + f_H \alpha w_H)r_H].$$

After rearrangement, we have the *intended* voluntary contribution:

$$\hat{r}_H = \frac{[\gamma(\delta_H, \alpha)(1 - \tau) - \mu^{-1}\tau]w_H}{\gamma(\delta_H, \alpha)(1 + f_H\alpha w_H) + (1 + \alpha)\mu^{-1}}.$$

This intention only materializes if $0 < \hat{r}_H \leq r_x$, and then there is no room for traditional savings.

In domain 3, the ceiling is too high to leave room for the traditional saving but is too low to have an interior optimal voluntary contribution: Since the consumption pair are

$$c_H = (1 - \tau - \alpha f_H r_x)w_H - r_x \quad \text{and} \quad d_H = \mu^{-1}[\tau + (1 + \alpha)r_x],$$

this situation occurs if and only if

$$\frac{[\gamma(\delta_H, 0)(1 - \tau) - \mu^{-1}\tau]w_H}{\gamma(\delta_H, 0)(1 + f_H\alpha w_H) + (1 + \alpha)\mu^{-1}} \leq r_x < \frac{[\gamma(\delta_H, \alpha)(1 - \tau) - \mu^{-1}\tau]w_H}{\gamma(\delta_H, \alpha)(1 + f_H\alpha w_H) + (1 + \alpha)\mu^{-1}}.$$

In domain 4, $r_H = r_x$ and there is room for traditional saving. Inserting into the general formula, yields the saving, which under normal conditions, cannot be negative:

$$s_H = \frac{\gamma(\delta_H, 0)(1 - \tau - f_H\alpha r_x - \mu^{-1}\tau)w_H - [\gamma(\delta_H, 0) + \mu^{-1}(1 + \alpha)]r_x}{\gamma(\delta_H, 0) + \mu^{-1}} \geq 0.$$

It is obvious that the bill of savers' 'perfection' is partly paid by the myopes:

$$c_L = \frac{w_L}{1 + \mu\gamma(\delta^\circ, 0)} - \alpha f_H r_H w_L < c_L^\circ \quad \text{and} \quad d_L = \frac{\gamma(\delta^\circ, 0)w_L}{1 + \mu\gamma(\delta^\circ, 0)} = d_L^\circ.$$

Symmetric system

It is much more promising to set such a low ceiling and such a high matching rate that the myopes just use up all their possibilities: *symmetric* system. Then $r_L = r_H = r_x$ and $\theta = \alpha r_x$, hence L's optimum condition

$$\mu^{-1}[\tau w_L + (1 + \alpha)r_x] = \gamma(\delta_L, \alpha)[w(1 - \tau - \alpha r_x) - r_x]$$

yields the ceiling:

$$r_x = \frac{\gamma(\delta_L, \alpha)(1 - \tau) - \mu^{-1}\tau}{\gamma(\delta_L, \alpha)(1 + w_L\alpha) + \mu^{-1}(1 + \alpha)}w_L.$$

For a high enough matching rate α , the ceiling r_x is positive and low enough to defend the pensioner L, without impoverishing the worker L. H's traditional saving is determined by $\gamma(\delta_H, \alpha)c_H = d_H$.

Otherwise, we can interpret this system as a composite mandatory system, where the mandatory contribution rate and the benefits are, respectively

$$\tau^* = \tau + \theta + r_L/w_i \quad \text{and} \quad b_i = \mu^{-1}[\tau w + (1 + \alpha)r_L + s_i], \quad i = L, H.$$

In words, a second pillar is added to the first, with a degressive contribution rate $\theta + r_L/w_i$ and a flat benefit $(1 + \alpha)r_L$. It is an empirical issue which packaging of the same system is more attractive: the fully mandatory or the one enlarged by a voluntary system?

Progressive mandatory benefit–proportional matching

Up to now we have demonstrated that in the combination of proportional mandatory benefit and proportional matching, it is debatable to subsidize the savers also from the taxes paid by the myopes, making the latter even poorer. We turn now to the analysis of the more complex cases, where the benefit or the matching function is piecewise (or inhomogeneously) linear. We keep our two types. Two cases are examined: (i) progressive mandatory pension with proportional matching, and (ii) proportional mandatory pension with progressive matching. Here we neglect the ceilings on voluntary contributions and subsidies.

We shall start the analysis with the case of progressive mandatory pension. Theoretically as well as empirically, most progressive pension systems can approximately be decomposed into a sum of a flat pension and a proportional pension, also taking care of the ceiling on mandatory contribution w_x (Disney, 2004):

$$b(w) = \min(\beta_0 + \beta w, b_x),$$

yielding the maximal mandatory pension: $b_x = \beta_0 + \beta w_x$. Apart from the ceiling, the mandatory pension balance is now $\tau = (\beta_0 + \beta)\mu$.

Indeed, a number of experts justify the existence of the tax-favored retirement accounts that such a system counterbalances the redistribution, inherent in a progressive mandatory pension system. For a large set of parameter values, we may assume that only H pays a voluntary contribution: $0 = r_L < r_H$.

The new pension formula contains a flat pension component $\beta_0 > 0$:

$$b_L = \beta_0 + \beta w_L \quad \text{and} \quad b_H = \beta_0 + \beta w_H.$$

The intended voluntary contribution is equal to

$$\hat{r}_H = \frac{[\gamma(\delta_H, \alpha)(1 - \tau) - \beta]w_H - \beta_0}{\gamma(\delta_H, \alpha)(1 + f_H \alpha w_H) + (1 + \alpha)\mu^{-1}}.$$

Finally, we provide the transfers received by type i in the mandatory and the voluntary pension systems, respectively: $T_{1i} = \mu b_i - \tau w_i$ and $T_{2i} = \alpha r_i - \theta w_i$.

According to Simonovits and Tóth (2009), a progressive mandatory pension system may diminish reported earnings, while the voluntary system may enhance it. This problem, however, is entirely neglected in the present paper.

Proportional mandatory benefit–progressive matching

Finally we investigate the case of a progressive matching function, we propose for Hungary. (The Czech voluntary pension system has a five-case piecewise matching–employee’s voluntary contribution function, starting with a rate of 0.5 and ending with a rate of 0.1. The ceiling is also quite low, about 4% of the total average wage. (For us it is immaterial that the Czech mandatory system is also progressive, almost flat.) A simplified, two-tier matching formula is as follows. Let $r_m \in (0, r_x)$ be the critical value

separating the higher (α_L) and the lower (α_H) matching rates, where $0 < \alpha_H < \alpha_L$. Then the progressive matching formula is given by

$$a(r) = \begin{cases} \alpha_L r & \text{if } 0 \leq r < r_m; \\ \alpha_L r_m + \alpha_H (r - r_m) & \text{if } r_m \leq r \leq r_x. \end{cases}$$

We confine our attention to the special case, where our parameter values are such that L chooses his voluntary contribution just at the threshold: $r_L = r_m$ and H chooses just the ceiling: $r_H = r_x$, and both types' traditional savings are zero.

We shall determine the matching rates so that L finds in his best interest to participate, and H's voluntary contribution does not reach the ceiling:

$$\alpha_L > \delta^o / \delta_L - 1 \quad \text{and} \quad \alpha_H \leq \max(1 / \delta_H - 1, 1).$$

Inserting the equations of c_i and d_i into the optimality conditions, and multiplying them by μ , yields:

$$\tau w_L + (1 + \alpha_L) r_L = \mu \gamma(\delta_L, \alpha) w_L (1 - \tau - \theta) - \mu \gamma(\delta_L, \alpha) r_L$$

and

$$\tau w_H + (\alpha_L - \alpha_H) r_L + (1 + \alpha_H) r_H = \mu \gamma(\delta_H, \alpha) w_H (1 - \tau - \theta) - \mu \gamma(\delta_H, \alpha) r_H.$$

Substituting the matching-voluntary contribution function into the definition of the earmarked tax rate

$$\theta = \alpha_L r_L + f_H \alpha_H (r_H - r_L) = (\alpha_L - f_H \alpha_H) r_L + f_H \alpha_H r_H$$

and rearranging the system of equations for r_L and r_H , we obtain the following 2×2 matrix \mathbf{E} of coefficients

$$e_{LL} = \mu \gamma(\delta_L, \alpha) [(\alpha_L - \alpha_H f_H) w_L + 1] + 1 + \alpha_L, \quad e_{LH} = \mu \gamma(\delta_L, \alpha) w_L f_H \alpha_H,$$

and

$$e_{HL} = \mu \gamma(\delta_H, \alpha) w_H (\alpha_L - f_H \alpha_H) + \alpha_L - \alpha_H, \quad e_{HH} = \mu \gamma(\delta_H, \alpha) (w_H f_H \alpha_H + 1) + 1 + \alpha_H,$$

and the following 2-vector \mathbf{g} with components

$$g_L = [\mu(1 - \tau) \gamma(\delta_L, \alpha) - \tau] w_L, \quad g_H = [\mu(1 - \tau) \gamma(\delta_H, \alpha) - \tau] w_H.$$

Using Cramer's rule, the two voluntary contributions are explicitly obtained from $\mathbf{E}\mathbf{r} = \mathbf{g}$. Under realistic assumptions, $r_H > r_L > 0$.

Since e_{LH} is the product of five positive numbers, each being much less than 1, while $e_{LL} > 1$, therefore e_{LH} is much less than e_{LL} . Thus we have the following approximation:

$$r_L \approx \frac{g_L}{e_{LL}} = \frac{[\mu(1 - \tau) \gamma(\delta_L, \alpha) - \tau] w_L}{\mu \gamma(\delta_L, \alpha) [(\alpha_L - \alpha_H f_H) w_L + 1] + 1 + \alpha_L}.$$

Again, we calculate the transfers arising in the tax-favored system: $T_{2L} = \alpha r_L - \theta w_L$, $T_{2H} = \alpha_L r_L + \alpha_H (r_H - r_L) - \theta w_L$. ($T_{1H} = T_{1L} = 0$.)

The special case, where the second matching rate is zero: $\alpha_H = 0$, deserves a special mentioning. Then we return to the proportional-symmetric proportional system, and the approximation above is exact, only the surplus of variable r_H over r_L becomes saving: $s_H = r_H - r_L$.

Two generalizations

Approaching the end of the analytical part, we outline two generalizations: (i) the distinction between high and low voluntary contributions in the asymmetric system and (ii) the exemption from tax on interest.

The first generalization allows for the distinction between actual and nominal members, leading to a *three-class* model: type HH is characterized by (w_H, δ_H) , while type HL is characterized by (w_H, δ_L) . The former pays a substantial voluntary contribution, the latter hardly pays anything. Of course, $f_{HH} + f_{HL} = f_H$ and $r_{HL} = 0$. We must apply now the proportional–asymmetric proportional system to the pair (HH,HL), but the balance $\theta = \alpha f_{HH} r_{HH}$ for the earmarked tax concerns three types.

The second generalization takes into account the fact that contrary to other savings, the accumulation of voluntary contributions and subsidies are tax exempt. Let κ be a real number between 0 and 1, showing the part of real value of traditional saving, that is taxed away. Therefore the pensioner’s consumption is equal to

$$d = b(w) + [r + a(r) + (1 - \kappa)s]/\mu.$$

Again, we confine our attention to the proportional–asymmetric proportional system. For a low enough ceiling r_x , the traditional saving is equal to

$$s_H = \frac{\gamma(\delta_H, -\kappa)(1 - \tau - f_H \alpha r_x - \beta)w_H - [\gamma(\delta_H, -\kappa) + \mu^{-1}(1 + \alpha)]r_x}{\gamma(\delta_H, -\kappa) + \kappa\mu^{-1}} \geq 0.$$

As a further simplification, let $r_x = 0$. In this limiting case,

$$s_H = \frac{[\gamma(\delta_H, -\kappa)(1 - \tau) - \mu^{-1}\tau]w_H}{\gamma(\delta_H, -\kappa) + \kappa\mu^{-1}} > 0$$

holds if and only if the relative loss κ due to the tax on interest is small enough. Substituting the definition of γ into s_H , the necessary and sufficient condition is

$$[\delta_H(1 - \kappa)]^{1/(1-\sigma)}(1 - \tau) > \mu^{-1}\tau, \quad \text{i.e.} \quad \kappa < 1 - \left(\frac{\mu^{-1}\tau}{1 - \tau}\right)^{1-\sigma} \frac{1}{\delta_H}.$$

Due to the complications of these generalizations, we shall only consider them numerically.

4. Numerical illustration

We continue our analysis with numerical illustrations. We assume that the time spent at retirement is half as long as that of working: $\mu = 0.5$. Basically we follow the logic of the previous section.

Discount factor and ceiling

In this Subsection, we assume that every worker has a unit total wage and we vary the discount factor and the ceiling on mandatory contributions.

As a baseline case, we calculate the optimal consumption pairs plus the mandatory contribution rate for four different discount factors. Each case has a name, two have an abbreviations: myope (L) and saver (H), and the other two have symbols: mean ($^{\circ}$) and government (*).

Table 1. *Discounting and optimal consumption pair: no matching*

Type i	Discounting factor δ_i	Worker c o n s u m p t i o n c_i	Pensioner d_i	Pension- saving rate τ_i
Myopic (L)	0.3	0.785	0.430	0.215
Mean ($^{\circ}$)	0.4	0.760	0.481	0.240
Saver (H)	0.5	0.739	0.522	0.261
Government (*)	1.0	0.667	0.667	0.333

Remark: $w = 1$.

Table 1 displays that the lower the discount factor, the higher is the worker consumption and the lower is the pensioner consumption, and the corresponding saving or mandatory contribution rate. (The value of the ratio depends on the exponent of the utility function, σ . The higher the absolute value of σ , the higher is the ratio of the pensioner's consumption to the worker's.)

As a detour, we shall only stay with the trivial model for a moment, where every worker's discount factor is the same and the government's is lower. Then the introduction of the tax-favored retirement accounts is quite transparent: for example, reducing the mandatory contribution rate $\tau = 0.24$ to $\tau' = 0.2$, and introducing a matching rate $\alpha = 0.3$, the voluntary contribution–wage coefficient $\rho = 0.036$ fixes the system. The actual total retirement saving is still 0.24.

From now on we move on to the two-type case, first with equal relative frequency $f_L = f_H = 1/2$ and different discount factors $\delta_L = 0.3$, $\delta_H = 0.5$ (first and third rows in Table 1). The government chooses a compromise: $\delta^{\circ} = 0.4$, i.e. the corresponding mandatory contribution rate $\tau = 0.24$ (second row in Table 1). To be short, we only experiment with two matching rates: 0.5 and 1.

Table 2 presents four values for the ceiling on the voluntary contribution: $r_x = 0, 0.02, 0.04$ and 0.06 . To save room, the constant $d_L = 0.481$ is omitted from Table 2. Although utility functions with negative values are acceptable, we transform them into positive values by adding a constant U_0 , namely $U_0 = 100$. Moreover U_i is multiplied by 10 at the display.

Table 2. *Proportional mandatory system–proportional matching*

Volunt- ary contr.'s ceiling r_x	Match- ing rate α	Worker cons.L c_L	Life- time util U_L	Intended voluntary contr. \hat{r}_H	Saving s_H	Worker consump-H c_H	Pensi- oner d_H	Life- time util U_H	Ear- marked tax rate θ
0.00	–	0.760	76.432	–	0.021	0.739	0.522	76.893	0.010
0.02	0.5	0.755	76.345	0.043	0.000	0.735	0.541	77.139	0.010
0.02	1.0	0.750	76.256	0.051	0.000	0.730	0.561	77.376	0.010
0.04	0.5	0.750	76.256	0.043	0.000	0.710	0.601	77.584	0.020
0.04	1.0	0.740	76.076	0.051	0.000	0.700	0.641	77.903	0.020
0.06	0.5	0.749	76.241	<i>0.043</i>	0.000	0.705	0.611	77.639	0.022
0.06	1.0	0.734	75.977	<i>0.051</i>	0.000	0.684	0.684	78.057	0.025

Remark: $w = 1$, $\tau = 0.24$ and $d_L = 0.481$.

In Table 2, increasing the matching rate α from 0.5 to 1 raises the value of the intended voluntary contribution \hat{r}_H from 0.043 to 0.051. In the first three cases of r_x , the intended voluntary contribution is higher than the ceiling, in the fourth, just the reverse. The realized intentions are *italicized*.

In Case 1, there is no matching: $r_x = 0$, thus the values of the matching rate and of the intended voluntary contribution are indifferent, we only display one of the two rows. The traditional saving is $s_H = 0.021$, quite large. In Case 2, $r_x = 0.02$. Here the pitfall of the matching system is already apparent: it punishes a little bit the myopes, and supports the savers, inciting them to pay a substantial voluntary contribution, while crowding out their traditional saving. The support for the savers and the punishment for the myopes are increased further. Case 3 is similar. In Case 4, the ceiling gives way to the intentions: for $r_x = 0.06$ and $\alpha = 1$, $r_H = 0.051$. The tax burden is simply $\theta = \alpha f_H r_H = 0.025$, quite large.

How does the total saving (i.e. the voluntary contribution and the traditional saving) change with the changes in the ceiling and the matching rate? When raising the ceiling, first ($r_x = 0.02$) the total saving is reduced, especially for the higher matching rate. (Incidentally, the traditional saving already disappears.) For higher ceiling, the total saving is increasing until reaching the intended voluntary contribution, and then stops growing any further.

It is noteworthy that—due to the utilitarian social welfare function—the welfare analysis depicts this process as advantageous: the decrease in the myopes' utility is overcompensated by the increase in the savers' utility. Of course, the more egalitarian social welfare function is assumed, the less probable is that this absurd result survives.

Higher paid versus lower paid

From now on we give up our assumption on equal earnings and work with heterogeneous earnings. To avoid too complex situations, which cannot be nicely presented, we identify

the savers with the higher paid and myopes with the lower paid. The former earn three times more than the latter: $w_H = 3w_L$. Since the higher paid are the savers, taking over Hungarian data, we put $f_H = 0.35$, implying $w_H \approx 1.765$, i.e. $f_L = 0.65$ and $w_L \approx 0.558$. Table 3 presents the five combinations analyzed in the theoretical part.

Table 3. *Mandatory and tax favored pensions: five combinations*

Earning w_i	Voluntary contri- bution r_i	Saving s_i	Worker c o n s u m p t i o n c_i	Pensioner d i o n d_i	Manda- t o r y t r a n s f e r T_{1i}	Volun- t a r y T_{2i}	Earmarked tax rate θ
1) proportional benefit–no matching							0
0.588	0	0	0.447	0.283	0	0	
1.765	0	0.021	1.304	0.922	0	0	
2) proportional benefit–asymmetric proportional matching							0.007
0.588	0	0	0.443	0.283	0	–0.004	
1.765	0.066	0	1.263	1.018	0	0.008	
3) proportional benefit–symmetric proportional matching							0.012
0.588	0.012	0	0.428	0.331	0	0.005	
1.765	0.012	0.010	1.297	0.917	0	–0.009	
4) progressive benefit–proportional matching							0.012
0.588	0.000	0	0.440	0.382	0.049	–0.007	
1.765	0.117	0	1.202	0.969	–0.092	0.013	
5) proportional benefit–progressive matching							0.015
0.588	0.012	0	0.426	0.330	0	0.003	
1.765	0.053	0	1.261	0.997	0	–0.005	

In Part 1 the mandatory contribution rate is fixed at $\tau = 0.24$ and the matching rate is taken as 0, therefore voluntary contribution is replaced by traditional saving. In our proportional mandatory system, the myopes are deemed to save mandatory more than they want, due to the mandatory contribution rate which is higher than their preferred value: $0.24 > 0.215$. In turn, the savers can save 2.1% of the average wage cost in addition to the mandatory saving. The myopes' old-age consumption becomes unacceptably low: a meager 28% of the average total wage.

In Part 2 we try to approximate the Hungarian situation. Note that the ceiling (in terms of average total wage cost) is $r_x = 750/2660 \approx 0.282$, the average voluntary contribution is about $\bar{r} = 67/2660 = 0.0252$. We assume that the members save equally and much below the ceiling: the case of (weak) asymmetry. Also we copy the matching rate 0.3 from recent Hungarian data, and neglect the ceiling. The numbers of the proportional mandatory and asymmetrically proportional voluntary system do not reflect the Hungarian reality well, since the Hungarian members only pay 3% rather than 6.6%. (Note the distance from the ceiling, which is 28%!) At least the value of the earmarked tax is well approximated: $\theta = 0.007$.

In the last two columns of Table 3 we display the transfers received by L or H from the mandatory and the voluntary systems, respectively. Since the mandatory systems of Parts 1, 2, 4 and 5 are proportional, the corresponding T_1 is identically zero. In the asymmetric voluntary system, the savers receive 0.8% of the average wage, while the myopes pay 0.4%. Moreover, nothing is achieved from the main objective of the voluntary system: L's old-age consumption remains as low as before.

In Part 3, the numbers of the proportional mandatory and symmetric proportional voluntary systems show our proposed solution. With a lifted matching rate $\alpha = 1$ the myopes pay a voluntary contribution $r_L = 0.012$ units, if the savers are constrained to pay that little, and then the earmarked tax rate is the same: $\theta = 0.012$. Then the transfer's direction will be just the opposite and the low-paid consumption rises to 0.331 units.

In Part 4 we illustrate the behavior in the progressive mandatory and the proportional voluntary systems. Half of the mandatory pension is flat, half is proportional: $\beta_0 = \beta = \tau/(2\mu) \approx 0.158$. There is a reasonable redistribution, since the mandatory contribution rate is equal to the myopes' and the mandatory system achieves a strong redistribution in their favor: $d_L = 0.382$. Though the myopes do not participate at the tax-favored system, they are more than compensated for the loss of 0.7% they suffer in the tax-favored system by the transfer of 4.9% received in the mandatory system.

Part 5 returns to the proportional mandatory pension, and assumes that the government subsidizes the tax-favored system progressively. The previous matching rate of 0.3 lies between the arbitrarily chosen two new rates: $\alpha_L = 1$ and $\alpha_H = 0.25$. The results in the combined proportional mandatory and progressive voluntary pension systems are reassuring: everybody pays some voluntary contribution, and the myopes' old-age consumption stays at 0.333. At the same time, the myopes hardly pay a voluntary contribution (0.012) and the savers' voluntary contribution is 4 times higher than the myopes'.

Next we present two examples for the welfare analysis. First we display the dependence of the welfare on the matching rate. We discuss the system studied in Part 2 of Table 3. Obviously, the optimal matching rate hardly differs from zero.

Table 4. *Optimal matching in the proportional–asymmetric proportional system*

Matching rate α	Earmarked tax		Voluntary contribution		Worker consumption	Pensioner consumption	Voluntary transfer	Social welfare
	rate θ	Earning w_i	contribution r_i	Worker c_i	Pensioner d_i	T_2	V	
0.00	0	0.588	0	0.447	0.283	0		
0.00	0	1.765	0.037	1.304	0.922	0	69.375	
<i>0.05</i>	<i>0.001</i>	<i>0.588</i>	<i>0</i>	<i>0.446</i>	<i>0.283</i>	<i>-0</i>		
<i>0.05</i>	<i>0.001</i>	<i>1.765</i>	<i>0.043</i>	<i>1.296</i>	<i>0.939</i>	<i>0.001</i>	<i>69.379</i>	
0.10	0.002	0.588	0	0.446	0.283	-0.001		
0.10	0.002	1.765	0.049	1.289	0.956	0.002	69.378	
0.15	0.003	0.588	0	0.445	0.283	-0.002		
0.15	0.003	1.765	0.054	1.282	0.972	0.003	69.372	
0.20	0.004	0.588	0	0.445	0.283	-0.002		
0.20	0.004	1.765	0.058	1.275	0.988	0.004	69.363	

The second piece of the welfare analysis concerns the combination of proportional mandatory and progressive voluntary systems (Part 4). As the computer shows, the social welfare is maximized at a very low value of α_L , thus we can safely return to the symmetric proportional voluntary system in Table 5. As a policy tool, we also consider a reduced mandatory contribution rate, namely $\tau = 0.2$.

Table 5. *Optimal matching in the proportional–symmetric proportional system*

Mandatory contribution rate τ	Earmarked tx		Voluntary contribution		Saving	Social welfare
	Matching rate α	rate θ	contribution r_L	Saving s_H	V	
0.20	0.2	0.015	0.089	0.001	69.132	
0.20	0.4	0.019	0.079	0.003	69.623	
0.20	0.6	0.022	0.070	0.005	70.010	
0.20	0.8	0.024	0.062	0.008	70.320	
<i>0.20</i>	<i>1.0</i>	<i>0.025</i>	<i>0.054</i>	<i>0.010</i>	<i>70.572</i>	
0.24	0.4	0.002	0.034	0.000	69.512	
0.24	0.6	0.007	0.026	0.002	69.865	
0.24	0.8	0.010	0.018	0.003	70.150	
0.24	1.0	0.012	0.011	0.005	70.382	

It can be seen that for a given mandatory contribution rate, a higher matching rate implies a higher welfare. For a given matching rate, lower mandatory contribution rate implies higher welfare. The optimal row is italicized again. Here we encounter the

danger that the savers can game the system and pay in the name of the others to collect the matching for themselves. The value of the earmarked tax rate is quite high: 2.5%. For a better understanding, three rows of Table 5 are further elaborated.

Table 6. *Details from the proportional-symmetric proportional system*

Contri- bution rate τ	Matching rate α	Earning w_i	Voluntary contribution r_i	Saving s_i	Worker consumption c_i	Pensioner consumption d_i
0.20	0.8	0.588	0.024	0	0.436	0.320
0.20	0.8	1.765	0.024	0.062	1.293	0.914
0.20	1.0	0.588	0.025	0	0.431	0.334
0.20	1.0	1.765	0.025	0.054	1.290	0.912
0.24	1.0	0.588	0.012	0	0.428	0.331
0.24	1.0	1.765	0.012	0.011	1.297	0.917

The middle pair of rows represent the social optimum. Its essence is the following: here the myopes' consumption (0.334) is greater than the counterpart of the lower matching rate and the same mandatory contribution rate (0.320) or the counterpart of the higher mandatory contribution rate and the same matching rate (0.331). This is induced by the common voluntary contribution $r_L = 0.012$, which is also complemented by the traditional saving $s_H = 0.011$ for the savers.

Finally, we make numerical illustrations for the two generalizations.

First, we consider the combination of a proportional mandatory and an asymmetric proportional voluntary systems and return to the previous matching rate $\alpha = 0.3$ and the earmarked tax rate θ . In our three-class model, the following parameter values assure the quasi maximum of the voluntary contribution: $f_{HH} = 0.05$ and $f_{HL} = 0.3$, with a raised discount factor $\delta_{HH} = 0.7$.

Table 7. *Proportional-strongly asymmetric proportional system*

Earning w_i	Discount factor δ_i	Voluntary contribution r_i	Worker c o n s u m p t i o n c_i	Pensioner consumption d_i	Voluntary transfer T_{2i}
0.588	0.3	0	0.446	0.283	-0.001
1.765	0.3	0	1.339	0.848	-0.003
1.765	0.7	0.120	1.217	1.161	0.033

Of course, in our strongly asymmetric model, only type HH pays a substantial voluntary contribution: 0.12 units instead of the previous 0.066 units. The actual ceiling 0.28 is still not reached.

We end our numerical illustrations with the study of the tax exemption. Following the logic of the text in the Introduction, the total gain on the interest tax on the accumulated assets is about 20%, i.e. $\kappa = 0.2$. To have a positive traditional saving at all, we must have $\kappa < (2 \times 0.24/0.76)^2/(0.5) = 0.202$, a very tiny slack.

5. Conclusions

We have constructed a model, where in addition to the contribution-based mandatory system, there is a tax-favored retirement system, financed from earmarked taxes. The voluntary contribution and the traditional saving are determined by the workers maximizing their subjective utility functions, while the corresponding earmarked tax rate is calculated by the government. In our “general equilibrium” model, we have done the first theoretical and numerical calculations. Reflecting stylized features of the current Hungarian system, our proportional tax-favored system is poorly targeted when the mandatory system is also proportional and generous: it helps just those who do not need this help. In the earlier, strongly progressive and modest mandatory system such a proportional matching might have been justified, only its extent was very excessive. In our opinion, a progressive voluntary system harmonizes with the proportional mandatory system, which moves the myopes in the direction of the savers. It is vital that the first matching rate should be much higher than the second one and the separating threshold be sufficiently low. A proportional–symmetric proportional system with a low ceiling can even yield a better solution. The results seem to be acceptable but a lot of further analytical arguments and numerical trials are needed to confirm our tentative deductions.

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