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INTER-FIRM COMPARISON AND DECOMPOSITION OF PRODUCTIVITY GAINS FOR REGULATORY PURPOSES*

In this study, temporal changes in firm-level productivity, referred to as “corporate productivity gains” are decomposed into causal factors, such as cost-saving technological changes, output growth in the presence of economies of scale, changes in input prices, and the effect of input price changes on the firm’s demand for inputs. The decomposition is then applied to inter-firm comparisons of productivity. Inter-firm differences in productivity gains are decomposed into the same causal factors as the productivity gains themselves. Following a brief description of the economic concepts and variables that are associated with the concept of productivity, an empirical study – a comparative analysis and decomposition of productivity gains in two real-life regulated companies – offers an opportunity to assess the practical problems of measurement and comparison, and to introduce some useful indexing and econometric tools. The empirical study demonstrates that inter-firm productivity comparison and decomposition can indeed be successfully achieved, and that they can play a useful role not only in corporate management, but also in the regulation of imperfect markets.

THE COMPARATIVE ANALYSIS OF PRODUCTIVITY

To a considerable extent, the operating conditions of companies that sell their products on imperfect, regulated markets are created, influenced and managed by the regulators themselves. This involves tremendous social responsibilities, which include not only the task of preventing regulated firms from rent-seeking anti-competitive practices and various other socially detrimental activities that may arise from their market power, privileges, etc., but also the duty of doing everything in their power to facilitate the efficient operation of the firms they regulate in order to ensure the supply of

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* I am indebted to three excellent colleagues and dear friends of mine, *Bernard J. Lefebvre*, *Robert E. Olley* and *Shafi A. Shaikh*. The present work draws from a major Canadian project, which was carried out under Olley’s chairmanship and my technical leadership. At the request of the Canadian Ministry of Communications, I investigated productivity comparisons within the framework of this project. I worked extensively with Shaikh on turning “dirty” accounting and managerial data into meaningful economic variables, and with Lefebvre on building various models of productivity decomposition. This article re-visits some of the results of my work. It emphasizes the usefulness of productivity comparisons and decomposition, hoping that it would help re-focus attention on the all-important but lately somewhat ignored topic of productivity, and introduce more quantitative analytical tools into economic analysis in my native Hungary.

regulated markets with the widest possible selection of products and services of the best possible quality at the lowest possible cost. These requirements apply to monopolies as well as any kind of imperfectly competitive markets. In order to fulfil their task, regulators are expected by society at large to have a thorough knowledge of not only the demand but also the supply side of the markets they regulate. The economic efficiency and financial well-being of regulated companies must be monitored, analysed, evaluated and corrected if necessary. The regulator cannot possibly accomplish this if it is not known – among other things – how productive the regulated firms are, how quickly their productivity improves over time, what the causes of their productivity gains are and how much improvement can reasonably be attributed to each cause. Productivity measurement and analysis are important regulatory tasks.

Managers of both regulated and unregulated companies also show a keen interest in productivity as one of the two main *endogenous* factors that determine corporate profits. It was recognised as early as before World War II that the size of, and changes in, corporate profit depend rather crucially on management decisions concerning changes of productivity and output prices. Productivity studies became one of the basic tools of short-term operational planning and budgeting as early as in the late 1960s. Productivity indices also became *ex ante* targets in addition to being *ex post* attributes of corporate performance. Budget- or plan-implicit productivity gains were derived to show how the fulfilment of the annual corporate budget or plan would improve productivity and profits. Assessments of the reality and reasonableness of the budget- or plan-implicit productivity gain resulted in annual productivity targets, and budgets or plans were modified if necessary to meet the productivity target. Simple measurements of actual annual productivity gains and *ad hoc* data analyses were no longer sufficient. Target setting required more knowledge. Corporate analysts were increasingly turning to sophisticated econometric models to understand the causes and consequences of productivity improvements.

Traditional managerial and regulatory knowledge of corporate productivity was based on simple index numbers showing annual changes in the relationship between the volumes of inputs and the outputs they were producing. Productivity measures were compared to each other in various ways. *Ad hoc* comparisons were made most often within the boundaries of a given firm, involving a comparison between its own past and present, or past and future, or present and future performance. These are the *intra-firm comparisons*. Comparisons with past performance are indispensably useful but may also be misleading. From a regulatory point of view, the greatest risk in comparing productivity performances over time is that the comparison may lead to an erroneous assessment of the economic performance or productive capability of a regulated firm. It has been a common problem throughout the history of regulation, that the regulator (and/or the management of the regulated firm) simply and mechanically assumed that the expected growth rate in the firm's productivity should be equal to some (usually the average) past growth rate. This is tantamount

to disregarding changes in operating conditions and their impact on productivity. When future conditions differ from past conditions, the productivity growth in the future will also differ from that in the past.

Productivity comparisons are often made with other firms. *Ad hoc inter-firm* comparisons can accomplish more than revealing where the productivity growth rate was higher, lower or the same in any given period. They may point to various causes and consequences of observed inter-firm differences but also carry a considerable risk of mistaken conclusions. Superficial comparisons are often built upon an implicit underlying assumption; *viz.*, if a comparable other firm has achieved a certain rate of output growth then a similar rate of improvement ought to be expected at one's own firm. This assumption may turn out to be correct or incorrect, but it is definitely harmful if the analysts carrying out the comparison do not explore in sufficient detail the factors affecting productivity, the differences between them and the effects of those differences.

While *ad hoc* comparisons may indeed direct the attention of the analyst toward some factors that affect productivity, they do not allow the quantification of their effects. Quantitative analyses of the thus identified effects cannot be made. For this reason, comparisons should not be made without proper decomposition. Decompositions not only *identify* the causes of improvements in productivity but also *quantify* their effects. Changes in productivity result from the combined effect of a number of economic variables such as growth in the output of the firm or cost saving technological changes. What makes the analysis particularly revealing and useful is that the total effect on productivity of each causal variable can be broken down into two components. One component is the magnitude or *rate of change* in the size of the variable. If output growth improves productivity then it will make a difference whether the output growth rate is 3 percent or 10 percent. Secondly, the variables that affect productivity produce their impact with certain *intensity*. The higher the intensity the greater the effect on productivity. Decompositions distinguish changes in the *size* of an explanatory variable from the *intensity* with which the explanatory (causal) variable influences productivity.

The econometric models that began to emerge in the 1970's facilitated a major new development in productivity analysis. This paper extends the method of analysing corporate productivity performance by using econometric models. It advances in two directions. First, a *joint multi-firm* econometric cost model is constructed for two (and possibly more) firms, whose productivity performances are to be compared. Second, using the estimated parameters of this joint cost model, a *causal decomposition* of inter-firm productivity differences is conducted, quantifying the inter-firm difference that is due to each causal factor. As we shall see in the next sections, the most important causes are cost-saving technological changes and the exploitation of economies of scale, when the volume of production increases.¹

¹ It is also possible to decompose temporal changes in productivity by *consequence*.

Decomposition also plays a key role in *forecasting* productivity. It is equally important for the regulator and the regulated firm to have an idea of how much improvement in productivity they can reasonably expect as a consequence of expected future operating conditions, or some specific forthcoming change in the operation of the firm. Mergers and acquisitions, among other things, typically generate various productivity-altering organisational and other changes in the operations of affected firms.

TEMPORAL CHANGES IN PRODUCTIVITY

We look at changes in corporate *total factor productivity*. As the name implies, this concept recognizes the firm’s output as the result of the combined productive services (inputs) of *all* of the firm’s factors of production. Changes in total factor productivity may occur in time or space. Temporal changes refer to progress in total factor productivity within a given firm between two points or periods in time. Temporal changes are usually referred to as *productivity gains*. Spatial changes, on the other hand, show the difference between the total factor productivities of two firms. Both types of changes are measured by *proportional volume changes*, defined as the natural logarithms of input and output *volume indices*.

Let us first define the variables used in the measurement of productivity!² The temporal *proportional change* of total factor productivity (the productivity gain) of a firm is defined as

$$\phi = \dot{q} - \dot{x}, \tag{1}$$

where (\dot{q}) denotes the temporal continuous proportional change in total output, and (\dot{x}) is the temporal continuous proportional change in total input. Temporally *continuous* proportional changes are expressed as Divisia volume indices. For the output, the time-continuous Divisia volume index is

$$\dot{q} = \sum_{i=1}^n \frac{P_i Q_i}{R} \dot{q}_i = \sum_{i=1}^n r_i \dot{q}_i, \tag{2}$$

where n outputs exist,

$\dot{q}_i = \frac{dQ_i}{dt} \frac{1}{Q_i} = d \ln Q_i$ is the temporal proportional change in the i -th output,

$R = \sum_{i=1}^n P_i Q_i$ is total revenue and r_i stands for the i -th output’s revenue share.

² In addition to total factor productivity, the concept of productivity may be expanded to the so-called *partial productivity* measures. These are termed “partial” because they show the relationship between the firm’s total output and *only one* category of its factor inputs. The most frequently investigated partial productivity measures are labour and capital productivity, but measures for material and sometimes even for other input categories also exist. Inter-firm comparisons and decompositions of partial productivity measures are not investigated in this article.

In practice, the analyst is forced to work with data that show *discrete* temporal changes between two points or periods in time. For this reason, a discrete approximation must be found for the time-continuous Divisia indices. The Törnqvist index is such an approximation. With discrete changes in outputs from a given period $(t - 1)$ to period t , the Törnqvist output volume index is

$$\dot{q}_i = \sum_{i=1}^n \bar{r}_{it} \ln \left(\frac{Q_{it}}{Q_{i,t-1}} \right), \quad (3)$$

where \bar{r} shows each i -th individual output's average revenue share, which is the simple arithmetic mean of its revenue shares in the two compared periods, that is

$$\bar{r}_{it} = \frac{r_{it} + r_{i,t-1}}{2}. \quad (4)$$

Inputs are treated in the same manner as outputs. The temporally continuous proportional change in total input is expressed by the Divisia volume index

$$\dot{x} = \sum_{j=1}^m \frac{W_j X_j}{C} \dot{x}_j = \sum_{j=1}^m s_j \dot{x}_j, \quad (5)$$

where m inputs exist,

$\dot{x}_j = \frac{dX_j}{dt} \frac{1}{X_j} = d \ln X_j$ is the temporal proportional change in the j -th input,

$C = \sum_{j=1}^m W_j X_j$ is total cost, and s_j denotes the j -th input's cost share.

Its discrete approximation, the Törnqvist input volume index describes the change in total input from period $(t - 1)$ to period t as

$$\dot{x}_t = \sum_{j=1}^m \bar{s}_{jt} \ln \left(\frac{X_{jt}}{X_{j,t-1}} \right), \quad (6)$$

where \bar{s} denotes the average cost share of the j -th input, that is³

$$\bar{s}_{jt} = \frac{s_{jt} + s_{j,t-1}}{2}. \quad (7)$$

As stated above, proportional volume changes are defined as the natural logarithms of input and output *volume indices*. This offers a choice of representation to the analyst. Indeed, some authors use index numbers, and not proportional changes, for the measurement of changes in input and output volumes. Expressed with the

³ For partial productivity measures, the Divisia and Törnqvist output volume indices remain the same as in equations (2) and (3), respectively, but the input volume indices (equations (5) and (6), respectively), must be re-defined. The W_j input prices and X_j input volumes (and consequently the \bar{s}_j cost shares and \dot{x}_j proportional input changes) refer only to the individual labour, capital, or material inputs that are included in the partial measure.

aid of volume indices, temporal change in the total factor productivity of the firm, the *productivity index*, becomes

$$\hat{\Phi} = \frac{\hat{Q}}{\hat{X}}, \quad (8)$$

where \hat{Q} and \hat{X} are the temporal volume indices of total output and total input, respectively. For discrete changes between the consecutive periods $(t - 1)$ and t , the Törnqvist volume indices of output and input are

$$\hat{Q}_t = \prod_{i=1}^n \left[\frac{Q_{it}}{Q_{i,t-1}} \right]^{\bar{r}_{it}}, \quad (9)$$

and

$$\hat{X}_t = \prod_{j=1}^m \left[\frac{X_{jt}}{X_{j,t-1}} \right]^{\bar{s}_{jt}}. \quad (10)$$

The Törnqvist volume indices of output and input are the weighted geometric means of the ratios of volume changes occurring in two consecutive periods in all outputs and in all inputs, respectively. The respective weights are the average revenue shares of individual outputs, and the average cost shares of individual inputs.

The temporal proportional change in total factor productivity, the *productivity gain* is defined as the natural logarithm of the *productivity index*; i.e.,

$$\hat{\phi} = \ln \hat{\Phi} = \ln \hat{Q} - \ln \hat{X}. \quad (11)$$

INTRA-FIRM DECOMPOSITION

Causal decompositions may be carried out within one firm. The phenomena that cause total factor productivity to change are identified and classified, and a certain part of the productivity gain is assigned to each cause. Successful identification of the causes requires extensive knowledge of certain basic economic characteristics of the firm's production process. Such knowledge may be derived from econometric production models.

Production functions, various forms of cost functions and profit functions may be selected as models. This study works with *total cost* functions. A cost function is a suitable analytical vehicle for estimating and describing the basic economic characteristics of the production process that are needed for productivity analysis. Production functions are not used here because they do not allow us to analyse the economic characteristics of multi-output production processes. Profit functions are omitted due to data problems.

For simplicity, and strictly for introductory purposes, intra-firm decomposition is attempted first with the aid of *single-output* cost functions. These are the simplest possible models we can use to demonstrate the essence of the decomposition in a transparent manner. Once we gain a basic insight, we switch to multi-output cost function specifications to gain a more detailed understanding of the various impacts on productivity of different categories of output.

The single-output case

It is common knowledge that a total cost function is capable of describing the production technology of a firm. The simplest general form of a total cost function assumes that a single output is produced, and that technological changes are exogenous; *i.e.*,

$$C = g(W_1, \dots, W_m, Q, T), \quad (12)$$

where C is the total economic cost of production, m inputs are employed by the firm, W denotes input prices, and T represents exogenous technological changes, whose measurement will be discussed later, when we describe our empirical models.

Technological changes cause increases in productivity by saving costs.⁴ The temporal productivity gain generated by technological changes, (\dot{B}), which we shall henceforth term the *technological effect*, is equal in size (but opposite in sign) to the temporal *shift* of the cost function generated by the technological changes,

$$\dot{B} = -\frac{\partial \ln C}{\partial t}. \quad (13)$$

This effect can be estimated using the elasticity of the total cost with respect to technological changes. The estimated cost function yields estimates of this elasticity, while the proportional change in technology can be expressed using data on technological changes.⁵

$$\dot{B} = -\left[1 - \frac{\partial \ln C}{\partial \ln T}\right] \frac{d \ln T}{dt} = \varepsilon_{CT} \dot{T}. \quad (14)$$

Output causes increases in productivity, when it grows in the presence of economies of scale.⁶ The temporal productivity gain generated by output growth, (\dot{E}), which we shall henceforth term the *output effect*, therefore depends on the degree of economies of scale and the growth rate of output. Economies of scale constitute a basic technological property of the firm. Their degree can be derived from the estimated total cost function, where it is the inverse of the output elasticity of cost ($\varepsilon_{CQ} = \varepsilon_{CQ}^{-1}$). Increases in the volume of output contribute to increases in productivity, when $0 < \varepsilon_{CQ} < 1$. The proportional change in output is calculated from the firm's output data. The output effect is therefore

$$\dot{E} = \left[1 - \frac{\partial \ln C}{\partial \ln Q}\right] \frac{d \ln Q}{dt} = \xi_{CQ} \dot{Q}, \quad (15)$$

where $\xi_{CQ} = 1 - \varepsilon_{CQ}$ and $\varepsilon_{CQ} = \partial \ln C / \partial \ln Q$ is the output elasticity of cost.⁷

⁴ The technological effect reflects the immediate and short-term cost-saving effects of technological changes. However, technological changes also may have long-term cost-saving effects, which gradually emerge over time with or without increases in the scale of production. We shall return to these effects when discussing estimation results.

⁵ Notice that $\varepsilon_{CT} = -\partial \ln C / \partial \ln T$. Since the elasticity is always negative, ε_{CT} is always positive if technological changes reduce cost.

⁶ Or it decreases in the presence of diseconomies of scale. This case, however, will not be discussed here.

⁷ Notice that $\xi_{CQ} = 1 - \partial \ln C / \partial \ln Q$ is positive when there are economies of scale; *i.e.*, when $0 < \partial \ln C / \partial \ln Q < 1$.

Equation (15) shows that the proportional temporal productivity change, the productivity gain, of a firm, represented by its total cost function, can be expressed as the sum of the products of 1. the temporal proportional change in each independent variable affecting productivity (and the cost) and 2. the cost elasticity with respect to the same independent variable. If the cost function is in the form of a regression equation containing an error variable, decomposition proceeds as follows:

$$\dot{\phi} = \varepsilon_{CT}\dot{T} + \xi_{CQ}\dot{q} + \dot{R}, \quad (16)$$

where \dot{R} represents the *residual* productivity change; *i.e.*, that portion of the observed (actual) productivity change, which is not explained by the first two expressions on the right-hand side of equation (16). This equation decomposes the change in productivity into causal components. This is the simplest form of decomposition. The productivity gain is caused by three effects: 1. technological effect, 2. output effect, and 3. residual effect:

$$\dot{\phi} = \dot{B} + \dot{E} + \dot{R}. \quad (17)$$

As stated above, in this simplest possible case, the firm's technology is described by a cost function with a single output and a single exogenous technological change. When technology is represented in a more elaborate fashion, in more detail and with greater precision, the decomposition of the productivity gain also becomes more elaborate. When it is recognized in the specification of the cost function, that the firm produces more than one output, instead of the single output effect shown in equation (17), as many output effects are generated as the number of outputs; *i.e.*,

$$\dot{\phi} = \dot{B} + \sum_{i=1}^n \dot{E}_i + \dot{R}, \quad (18)$$

where the individual output effects \dot{E}_i appear as the product of 1. the proportional change in output i and 2. the cost elasticity with respect to the same output i ,

$$\dot{E}_i = \xi_{CQi}\dot{q}_i. \quad (19)$$

The explicit inclusion of complex technological changes in the model complicates the decomposition of productivity gains in a variety of ways, especially if input-neutral (Hicksian) technological changes appear contemporaneously with changes that affect the input structure and/or result in input or output augmentation. Such complex models are outside the scope of this study.

The multi-output case

Most firms produce more than one product or service. When there is more than one output, changes in the output structure presumably have an effect on productivity. In order to be able to estimate the effects of output structure in addition to the effects discussed above, a multi-output cost model must be introduced into the

analysis. The single output in the cost function in equation (12) is now replaced by n number of individual output variables:

$$C = g(W_1, \dots, W_m, Q_1, \dots, Q_n; T). \tag{20}$$

By taking the total time derivative of this cost function, introducing Shepard's lemma whereby $\partial g/\partial W_j = X_j$, and rearranging the resulting equation, we obtain

$$\dot{B} = \sum_{i=1}^n \varepsilon_{CQ_i} \dot{Q}_i - \sum_{j=1}^m s_j \dot{x}_j, \tag{21}$$

where \dot{B} expresses the extent of the shift in the cost function that is caused by the technological change, and $\varepsilon_{CQ_i} = \frac{\partial g}{\partial Q_i} \frac{Q_i}{C} = \frac{\partial \ln C}{\partial \ln Q_i}$ denotes the elasticity of cost with respect to the i -th output.

Comparing equation (21) with the Divisia indices in equations (1), (2) and (5), it becomes obvious that the temporal cost shift expresses the productivity gain without distortion if $\varepsilon_{CQ_i} = r_i \forall i$, that is, when the revenue share of each output equals the cost elasticity with respect to the same output, resulting in

$$\frac{P_i Q_i}{R} = \frac{\partial g}{\partial Q_i} \frac{Q_i}{C}. \tag{22}$$

This equality can materialise only if 1. the price of each output equals its marginal cost ($P_i = MC_i = \partial g/\partial Q_i, \forall i$), and 2. total revenue is equal to total cost ($R = C$). Both conditions are met if the production process is characterised by constant returns to scale and if the input and output markets of the firm are perfectly competitive. The output markets of regulated firms cannot, however, be said to be perfectly competitive. Furthermore, regulated firms tend to be engaged in network-based production, therefore their technology may well be characterized by economies of scale. It is precisely these properties that warrant their regulation. For this reason, the productivity gains of regulated firms may differ in size from the temporal proportional shifts in their cost. Using equation (22), the difference can be expressed as

$$\dot{\phi} - \dot{B} = \sum_{i=1}^n \left[\frac{P_i Q_i}{R} - \frac{MC_i Q_i}{C} \right] \dot{Q}_i. \tag{23}$$

The productivity gain can be expressed in a very instructive way from equation (23) if we add and subtract $(P_i Q_i)/C$, and rearrange the right-hand side of the equation.⁸ We obtain

$$\dot{\phi} = \sum_{i=1}^n \left[(P_i - MC_i) \frac{Q_i}{C} \right] \dot{Q}_i + \sum_{i=1}^n [(P_i Q_i)(R^{-1} - C^{-1})] \cdot \dot{Q}_i + \dot{B}. \tag{24}$$

⁸ The consequences of non-marginal cost pricing were first analysed by Denny, Fuss & Everson [1979].

For a multi-output model, this is the starting point of the decomposition of productivity gains. The equation demonstrates that the productivity gain equals the change in cost ($\dot{\phi} - \dot{B}$) if the firm has constant returns to scale and marginal cost pricing. $MC_i = 0$, but $R - C < 0$ if the firm has marginal cost pricing but there are economies of scale. $P_i - MC_i \neq 0$, but $R = C$ if the firm has some sort of cost-covering zero-profit constraint, as in the case of average cost pricing or Ramsey pricing. The first and second items on the right-hand side of the equation are activated if there are economies of scale and – for this reason or independently from this – prices do not equal marginal costs.

The concept of average cost may be meaningful for some firms if the great majority of their production costs are output specific. If average cost is meaningful then useful information can be generated by decomposing the *price – marginal cost* difference into a *price – average cost* and an *average cost – marginal cost* difference; i.e.,

$$P_i - MC_i = (P_i - AC_i) + (AC_i - MC_i). \tag{25}$$

Substituting equation (25) into equation (24) the following formula is obtained:

$$\begin{aligned} \dot{\phi} = & \sum_{i=1}^n \left[(P_i - AC_i) \frac{Q_i}{C} \right] \cdot \dot{q}_i + \sum_{i=1}^n \left[(AC_i - MC_i) \frac{Q_i}{C} \right] \cdot \dot{q}_i + \\ & + \sum_{i=1}^n [(P_i Q_i)(R^{-1} - C^{-1})] \cdot \dot{q}_i + \dot{B}. \end{aligned} \tag{26}$$

The rather lengthy and complicated structure of this equation may be simplified by replacing the three multipliers of \dot{q}_i on the right-hand side by ZA_i, ZM_i and ZR_i , as in

$$\dot{\phi} = \sum_i ZA_i \dot{q}_i + \sum_i ZM_i \dot{q}_i + \sum_i ZR_i \dot{q}_i + \dot{B}. \tag{27}$$

SPATIAL CHANGES IN PRODUCTIVITY

As suggested in the introductory section, there are two kinds of inter-firm productivity comparisons. On the one hand, the productivity “levels” of two firms may be compared by composing spatial volume indices, showing which one is more productive and quantifying the difference between their productivity “levels”. On the other hand, each of the compared firms has a time series of temporal productivity gains, and these can be compared as well by composing temporal volume indices for both, showing which one improves productivity faster and quantifying the relationship between their “speeds”. The first case is that of *level comparison*, and the second case is that of *speed comparison*.

Let us take a quick look at the spatial indices! The spatial index of productivity is defined analogously with the definition of the temporal index. The only differ-

ence is that in a spatial index, the price and volume data for periods $(t - 1)$ and t are replaced by the data obtained for Firms A and B . Thus, the spatial proportional change (difference) in productivity is

$$\begin{aligned} \dot{\phi}_{AB} = & \sum_{i=1}^n \frac{1}{2} \left[\frac{P_{iA} Q_{iA}}{\sum_{i=1}^n P_{iA} Q_{iA}} + \frac{P_{iB} Q_{iB}}{\sum_{i=1}^n P_{iB} Q_{iB}} \right] \ln \left(\frac{Q_{iA}}{Q_{iB}} \right) - \\ & - \sum_{j=1}^m \frac{1}{2} \left[\frac{W_{jA} X_{jA}}{\sum_{j=1}^m W_{jA} X_{jA}} + \frac{W_{jB} X_{jB}}{\sum_{j=1}^m W_{jB} X_{jB}} \right] \ln \left(\frac{X_{jA}}{X_{jB}} \right). \end{aligned} \quad (28)$$

The measurement of spatial differences in productivity and their inter-firm decomposition are outside the scope of this study.⁹ Our task is limited to “speed comparisons” and the decomposition of inter-firm differences in temporal productivity gains.

The difference between the productivity gains of two firms (A and B) can be expressed as

$$\Delta \dot{\phi}_t = \dot{\phi}_{At} - \dot{\phi}_{Bt} = (\dot{q}_{At} - \dot{x}_{At}) - (\dot{q}_{Bt} - \dot{x}_{Bt}) = \Delta \dot{q}_t - \Delta \dot{x}_t, \quad (29)$$

where t refers to productivity gain in period t , showing change relative to $t-1$.

INTER-FIRM DECOMPOSITION

Equation (16) demonstrates the simplest decomposition of productivity gains. It shows that temporal increases in productivity are generated by three effects: 1. a technology effect, 2. an output effect and 3. a residual effect. When two firms, A and B , are compared, this kind of decomposition can be performed for the temporal productivity gains of both firms. Furthermore, it is also possible to *decompose the differences* in the productivity gains of the two firms as

$$\Delta \dot{\phi}_t = \dot{\phi}_A - \dot{\phi}_B = (\dot{B}_A - \dot{B}_B) + (\dot{E}_A - \dot{E}_B) + (\dot{R}_A - \dot{R}_B) = \Delta \dot{B} + \Delta \dot{E} + \Delta \dot{R}. \quad (30)$$

Temporal productivity gains always refer to specified time periods, such as years. In this study we use annual data. The productivity *gain* of year t is understood as the productivity *level* in year t , expressed as a change over the productivity *level* in the previous year $t - 1$. The comparison and decomposition are also done annually. However, for simplicity, the references to time have been omitted from the following lengthy decomposition equations of this section.

Just as the Divisia indices that assume temporally continuous changes were approximated for discrete changes by Törnqvist indices, we once again need

⁹ The spatial comparison of productivity “levels” of firms raises many severe practical problems. It is very difficult, sometimes impossible, to ensure consistency and comparability for the technology, input and output data of different firms. Kiss [1984] discussed some of the problems of measurement and comparison.

a discrete approximation of the continuous changes assumed in equation (30). *Kiss* [1981], [1983] discussed some issues of discrete approximations. The simplest possible solution is recommended in the current study. For differences in technological effects, the equation becomes

$$\dot{B}_A - \dot{B}_B = \bar{T}\Delta\varepsilon_{CT} + \bar{\varepsilon}_{CT}\Delta\dot{T} = \frac{(\dot{T}_A + \dot{T}_B)}{2}(\varepsilon_{CTA} - \varepsilon_{CTB}) + \frac{(\varepsilon_{CTA} + \varepsilon_{CTB})}{2}(\dot{T}_A - \dot{T}_B). \quad (31)$$

We approximate the difference in output effects in a similar way; *i.e.*,

$$\dot{E}_A - \dot{E}_B = \bar{Q}\Delta\xi_{CQ} + \bar{\xi}_{CQ}\Delta\dot{Q} = \frac{(\dot{Q}_A + \dot{Q}_B)}{2}(\xi_{CQA} - \xi_{CQB}) + \frac{(\xi_{CQA} + \xi_{CQB})}{2}(\dot{Q}_A - \dot{Q}_B). \quad (32)$$

Changes in input prices also play a role in the productivity performance of firms by influencing the firm's demand for factor inputs. Let us investigate this role! *Temporal* proportional changes in individual inputs (such as labour, capital and materials) can be decomposed into causal components in a similar fashion. The decomposition of change in the j -th input is

$$\dot{X}_{jt} = \varepsilon_{jT}\dot{T} + \varepsilon_{jQ}\dot{Q} + \sum_{i=1}^m \varepsilon_{ji}\dot{W}_i + \dot{R}_j, \quad (33)$$

where ε_{jT} , ε_{jQ} and ε_{ji} show the elasticity of the j -th input with respect to technology, output and the i -th input price, respectively, and $\dot{W}_i = d \ln W_i / dt$ denotes the proportional change in the i -th input's price. Simplifying equation (33), we obtain equation (34) as

$$\dot{X}_j = \dot{B}_j + \dot{E}_j + \sum \dot{W}_j + \dot{R}_j. \quad (34)$$

In this case, the *spatial* decomposition of inter-firm difference becomes

$$\begin{aligned} \Delta\dot{X}_j = \dot{X}_{jA} - \dot{X}_{jB} &= (\dot{B}_{jA} - \dot{B}_{jB}) + (\dot{E}_{jA} - \dot{E}_{jB}) + (\sum \dot{W}_{jA} - \sum \dot{W}_{jB}) + \\ &+ (\dot{R}_{jA} - \dot{R}_{jB}) = \Delta\dot{B}_j + \Delta\dot{E}_j + \sum \dot{W}_j + \Delta\dot{R}_j. \end{aligned} \quad (35)$$

This equation shows that the difference between the two firms with respect to the proportional change in the volume of the j -th input can be decomposed into 1. a technological effect, 2. an output effect, 3. an input-price effect and 4. a residual effect. The input-price effect $\Delta\sum \dot{W}_j = (\sum \dot{W}_{jA} - \sum \dot{W}_{jB})$ can be further divided into an own-price effect $\varepsilon_{jj}\dot{W}_j$ and $(m - 1)$ number of cross-price effects (when $j \neq i$).

Now we return to productivity. It has been shown that if the decomposition of temporal proportional changes in productivity as defined by equation (27) can be performed for both firms, the two temporal decompositions can be used to identify the inter-firm differences in causal components. Where there is more than one output, the first three additive factors on the right-hand side of equation (27) must be repeated for each output. Assuming for simplicity that both companies produce two outputs (α and β) and using equation (27) as a point of departure, the inter-firm decomposition of the sources of temporal changes takes the following, rather lengthy, form:

$$\begin{aligned} \Delta\dot{\phi} = \dot{\phi}_A - \dot{\phi}_B = & (ZA_{\alpha A}\dot{q}_{\alpha A} - ZA_{\alpha B}\dot{q}_{\alpha B}) + (ZM_{\alpha A}\dot{q}_{\alpha A} - ZM_{\alpha B}\dot{q}_{\alpha B}) + \\ & + (ZA_{\beta A}\dot{q}_{\beta A} - ZA_{\beta B}\dot{q}_{\beta B}) + (ZM_{\beta A}\dot{q}_{\beta A} - ZM_{\beta B}\dot{q}_{\beta B}) \\ & + (ZR_{\alpha A}\dot{q}_{\alpha A} - ZR_{\alpha B}\dot{q}_{\alpha B}) + (ZR_{\beta A}\dot{q}_{\beta A} - ZR_{\beta B}\dot{q}_{\beta B}) + (\dot{B}_A - \dot{B}_B) \end{aligned} \quad (36)$$

Equation (31) is particularly instructive because it shows how the $\dot{B}_A - \dot{B}_B$ difference can be decomposed into two causal components: 1. the difference between the proportionate changes in the variables and 2. the difference between their cost elasticities. The same decomposition can be performed for the remaining differences. The seven differences contained in equation (36) thus decompose the inter-firm difference in productivity gain into a total of nine well-defined and clearly characterised components. The tenth component is the inter-firm difference due to the unexplainable residual effect. The categories emerging from the decomposition are summarised in *Table 1*.

TABLE 1 • Components of inter-firm differences in productivity gains

Explanatory factor	Formula
1. An increase in the production of output α	$\Delta\dot{q}_\alpha(\bar{Z}A_\alpha + \bar{Z}M_\alpha + \bar{Z}R_\alpha)$
2. Economies of scale specific to output α	$\Delta ZM_\alpha \bar{q}_\alpha$
3. The non-average-cost price of output α	$\Delta ZA_\alpha \bar{q}_\alpha$
4. Increase in the production of output β	$\Delta\dot{q}_\beta(\bar{Z}A_\beta + \bar{Z}M_\beta + \bar{Z}R_\beta)$
5. Economies of scale specific to output β	$\Delta ZM_\beta \bar{q}_\beta$
6. The non-average-cost price of output β	$\Delta ZA_\beta \bar{q}_\beta$
7. Profit/loss due to non-cost-covering pricing	$\Delta ZR_\alpha \bar{q}_\alpha + \Delta ZR_\beta \bar{q}_\beta$
8. Technological changes	$\Delta \bar{T} \bar{\varepsilon}_{CT}$
9. Technology elasticity of cost	$\Delta \varepsilon_{CT} \bar{T}$
10. Residual increase in productivity	—

Changes in the output structure of Firm *A* may differ from the changes in the output structure of Firm *B*. The differences result from the diverging growth rates of outputs α and β in the two companies. Their effects are shown in Items 1 and 4 of the table. We may, however, also give a broader interpretation to the effects of changes in output structure. The first three items show the total effect of output α on inter-firm differences of productivity gains, while the next three items give the same information for output β . Item 7 equals zero if the two firms use cost-covering average cost pricing. If the prices of one of the outputs exceed its average cost and generate surplus revenue beyond the cost while the prices of the other output are lower than the average cost and therefore generate a loss, *i.e.*, internal cross-subsidisation takes place, the total effect of non-average-cost pricing will be the sum of Items 3, 6 and 7. With two outputs, this may be very simple; *e.g.*, one output – say β – may generate a profit while the other output – say α – may generate a loss for both firms. If we look at Firm *A* or Firm *B* separately, the loss effect

is due to the non-average-cost prices of output α , and the profit effect is due to the non-average-cost prices of output β . The definitions used for the decomposition can be modified so that they reflect this situation. However, we cannot “allocate” the profit-loss effect if more than two outputs exist, unless we obtain more information than what can be reasonably assumed to be available, and construct a more complex multi-output model.

A REPRESENTATIVE INTER-FIRM COMPARISON¹⁰

In the remainder of this article it is demonstrated – with the aid of *actual firm-level data* on output and input prices and volumes as well as technological changes – how inter-firm comparisons and decompositions of annual productivity gains can be successfully conducted. Two firms have been chosen for the empirical study, mainly because their technologies were sufficiently similar, and their data were publicly available. Since the purpose of this demonstration is the illustration of the process, their names, locations, and the chosen period of observation are not revealed.

Let us introduce the data! The two companies make the same products. The output volume of Firm *B* surpassed that of Firm *A* by a great deal but *A*'s growth rate (14 percent per annum on average) was substantially higher than *B*'s (8 percent per annum on average). A relatively fast process of catching up is witnessed. The output growth rates are displayed in Table *A1* in the *Appendix*. At the start of the observation period, Output α had a 33 percent revenue share in Firm *A*, and 57 percent in Firm *B*. During the observation period, this revenue share declined to 30 percent in Firm *A* and to 48 percent in Firm *B*, while the share of Output β increased from 62 percent to 66 percent in Firm *A*, and from 37 percent to 48 percent in Firm *B*, demonstrating that Firm *B* underwent a faster structural change than Firm *A*. The third Output g had low revenue shares, which did not change significantly over time; it decreased from 5 percent to 4 percent in both companies. Most of the output effects were therefore generated by Outputs β and α . *Figure A1* in the *Appendix* shows that very different forces acted upon the markets of the two companies. During the 12-year period of observation, there were only seven years when the growth of the output of the two companies accelerated or decelerated in parallel.

A phenomenon of some importance with respect to productivity performance is that the faster growth of Firm *A* was accompanied by greater annual fluctuations. The standard deviation of the former (10.7) is almost twice as large as that of the

¹⁰ In order to maintain focus on the problems and solutions associated with concepts, measurements and analytical tools that regulatory and corporate productivity analysts encounter in their normal practice, we strive to divert attention from the representative firms themselves. They are just an illustration. This paper is not about them; it is about the principles, methods, and the practice of productivity analysis.

latter (5.8). Large variability in the growth rates of the outputs was accompanied by an almost equally large variability in the growth rates of the inputs. The input growth rates are shown in *Table A2* in the *Appendix*. Their standard deviation was 10.1 for Firm *A* and 4.7 for Firm *B*. The capital inputs of both companies displayed a linear increase over time but there was substantial fluctuation in the growth of material input and, for Firm *A*, in the growth of labour input as well. As revealed by *Figure A2* in the *Appendix*, not only the output markets but also the input markets of the two companies were affected by quite different forces. During the 12-year period, there were only four years when the input growth of the two companies accelerated or decelerated in parallel.

The high annual fluctuations in outputs and inputs do not lead to high fluctuations in the annual productivity gains if there is a strong correlation between them. *Figures A3* and *A4*, however, show a weak correlation for both companies. During the 12-year period, the increase in output and input accelerated or decelerated simultaneously in Firm *A* and Firm *B* in only five of the years. Consequently, as shown in *Figure A5*, the annual productivity growth rates are characterised by a great deal of fluctuation. The annual proportional changes in productivity are displayed in *Table 2* below and in *Figure A4* in the *Appendix*. As an illustration of the degree of fluctuations, the table also shows the extent of deviation from the mean.

Technological changes presumably did not influence the annual fluctuations in productivity gains to a significant extent. There are two reasons for this: the first one applies to Firm *B* and the second one mainly to Firm *A*. First, technological changes took place at a fast pace but were relatively evenly distributed in time for Firm *B* (see

TABLE 2 • Changes in productivity in Firm *A* and Firm *B* (100 ϕ_t)

Year (<i>t</i>)	Annual proportional change			Deviation from the mean		
	<i>A</i>	<i>B</i>	<i>A</i> – <i>B</i>	<i>A</i>	<i>B</i>	<i>A</i> – <i>B</i>
1	9.39	3.54	5.85	-3.04	0.28	-3.33
2	3.72	4.20	-0.48	2.63	-0.38	3.00
3	3.61	0.47	3.14	2.74	3.35	-0.62
4	8.78	6.07	2.71	-2.43	-2.25	-0.19
5	6.70	5.17	1.53	-0.35	-1.35	0.99
6	12.67	5.65	7.02	-6.32	-1.83	-4.50
7	1.60	8.45	-6.85	4.75	-4.63	9.37
8	-1.42	2.33	-3.75	7.77	1.49	6.27
9	3.77	1.14	2.63	2.58	2.68	-0.11
10	8.61	2.36	6.25	-2.26	1.46	-3.73
11	9.70	3.49	6.21	-3.35	0.33	-3.69
12	9.47	3.93	5.54	-3.12	-0.11	-3.02
13	5.89	2.90	2.99	0.46	0.92	-0.47
Mean	6.35	3.82	2.52	0.00	0.00	0.00

Table A3 and *Figure A10*). Their standard deviation has a low value for this firm. The same observation however would not apply to Firm *A*, where the technology variable shows high annual fluctuation and a large standard deviation. Second, as *Figure A8* indicates, there is a weak correlation between annual changes in technology and productivity for Firm *A*. This suggests that technological changes did not have significant immediate or short-term cost-saving effects. *Figure A9* shows a stronger correlation pointing to more intensive effects of technological changes in Firm *B*.

There are two important further observations. First, the introduction new technologies was completed much faster in Firm *A* than in Firm *B*. Faster output growth rate and better financial position appear to have played a role in this. Second, although technological progress and output growth show the same cyclic behaviour, the technology cycles follow the output cycles with some delay.

SPECIFICATION OF THE EMPIRICAL MODELS

Initially the technologies of the two firms are analysed separately in *firm-specific* production models. Results from the models that are specified and estimated independently for Firm *A* and Firm *B* are expected to provide guidance for an assessment of whether the statistical quality and economic meaningfulness of the parameter estimates can be improved by building *shared models*. If – as in our case – results from the estimated firm-specific models suggest that there is a need and room for significant improvement, shared models are attempted.

A thorough investigation of technological properties and management behaviour suggested that the production processes of both firms could be expressed with the aid of either the single-output or the multi-output total cost functions of equations (12) and (20), respectively.

As mentioned before, the two companies were producing the same three outputs. However, as a first approximation, it was assumed for the sake of simplicity that only one output – an aggregate of the three – was produced. The simplicity of the single output model makes it especially suitable for illustrating the basic characteristics of comparisons and decompositions. This assumption was discarded in our second approximation, and a multi-output model was built.

Equations (12) and (20) are capable of describing either a cost minimising or a profit maximising firm. The initial set of cost models was based on the assumption of pure cost minimising corporate behaviour. Cost minimisers endogenously determine input volumes in order to produce exogenously determined volumes of output volumes at minimum cost, subject to exogenous input prices. The exogeneity of input prices was a reasonable assumption because both firms purchased their inputs on markets which could safely be characterised as perfectly competitive. Their output on the other hand was neither purely exogenous nor purely endogenous.

On the one hand, regulation imposed on both firms some obligations to satisfy demand generated by prices that had been strongly influenced by regulation, thereby *de facto* defining or at least strongly influencing market size. On the other hand, due to their market power and the light-handed nature of regulation, the management of both firms could influence output prices and volumes to a considerable degree. Recognising the resulting endogeneity of outputs, and as an alternative to pure cost minimisation, profit maximising corporate behaviour, subject to endogenous output volumes, was also assumed in the second set of cost models.¹¹

Technological changes were assumed exogenous in both sets. The assumption of exogeneity seems essentially reasonable. The main driving force of the observed technological changes was the digital revolution itself, which left very limited technological choices for the companies. It was clear to both firms that their business success was to a large extent a function of how rapidly and efficiently they managed to exploit the constantly emerging new technological possibilities.¹²

The total cost function is specified as a so-called *transcendental* function. It is generated by the flexible, second-order Taylor series local expansion of the general-form neoclassical cost function. The transcendental specification has been selected in order to avoid *a priori* constraints on technological properties. The mathematical shape of a transcendental function is determined by the data rather than by such constraints.

First we assume a single homogeneous output, include the prices of three homogeneous inputs (labour, capital and materials), and apply a temporal index series of cost-saving exogenous technological changes. The firm-specific transcendental cost function relying on these assumptions is

$$\begin{aligned}
 C = & \alpha_0 + \alpha_1 w + \alpha_2 r + \alpha_3 m + \alpha_4 Q + \beta T + 1/2(\gamma_{11} w^2 + \gamma_{22} r^2 \\
 & + \gamma_{33} m^2 + \gamma_{44} Q^2 + \beta_T T^2) + \gamma_{12} wr + \gamma_{13} wm + \gamma_{23} rm + \gamma_{14} wQ + \\
 & + \gamma_{24} rQ + \gamma_{34} mQ + \beta_1 wT + \beta_2 rT + \beta_3 mT + \beta_Q QT,
 \end{aligned}
 \tag{37}$$

where C denotes the total economic cost of production; w , r and m are the prices of labour, capital and material inputs, respectively; Q represents the volume of the single output; and T denotes the technological index.

The usual parametric restrictions to impose first-order homogeneity in input prices are applied.¹³ Following the usual practice of econometric cost analyses, under the assumption of cost minimisation the cost function is estimated as part of

¹¹ The profit maximising cost models have been excluded from this paper.

¹² Since certain elements of technological changes are highly dependent on management decisions, such elements should be regarded as endogenous. However, due to limitations in size, this study is not extended to more complex, detailed treatments of technological changes.

¹³ The imposition of $\sum_j \alpha_j = 1$; $\sum_j \gamma_{ji} = \sum_j \beta_j = 0$ ($i = 1, \dots, 4$; $j = 1, \dots, 3$) results in first-degree homogeneity of the cost function in input prices. This common sense requirement ensures that if all input prices are raised by the same percentage then production cost undergoes an identical percentage increase.

a simultaneous equation system in which the application of Shephard's well-known lemma yields two cost share equations.¹⁴ The parameters are estimated in a modified version of the procedure originally developed by Zellner [1962] for the estimation of "seemingly unrelated" regression equations.

The productivity performances of any two companies may be compared to each other in order to establish which one has higher or faster improving productivity. Firm-specific cost models may also reveal what causes generated how much cost saving and improvement in productivity. Inter-firm comparisons can be made. It is not required that technological or other similarities exist between the two firms. The quality of the estimated parameters can, however, often be improved in a situation where technological and other similarities between the compared firms allow the building of a *common technology model*. When relatively few observations are available, for instance, the use of this model increases the degrees of freedom and thus contributes to the "sharpening" of the parameter estimates. Building a common technology model often leads to a significant improvement, when the two companies operate in the same industry.

Common technology is represented by a *common or "shared" total cost function* which allows both similarities and differences to exist between the two firms. Technological similarities are revealed by forming and testing a set of constraining null hypotheses that express various equivalences between their technologies. The test results allow us to describe a technology some parts of which are shared by the two companies, while other parts are not. The least constrained shared cost function allows differences between Firm A and Firm B in each of the parameters of the cost function, but assumes that the same variance-covariance matrix applies to both companies. Binary dummy variables allow each parameter of the shared cost function to be firm-specific. The use of all possible dummies ($D_A = 1, D_B = 0$) results in the following lengthy specification:

$$\begin{aligned}
 C = & \alpha_0 + \alpha_{0A}D_A + (\alpha_1 + \alpha_{1A}D_A)w + (\alpha_2 + \alpha_{2A}D_A)r + (\alpha_3 + \alpha_{3A}D_A)m + & (38) \\
 & + (\alpha_4 + \alpha_{4A}D_A)Q + (\beta + \beta_A D_A)T + 1/2((\gamma_{11} + \gamma_{11A}D_A)w^2 + (\gamma_{22} + \gamma_{22A}D_A)r^2 + \\
 & + (\gamma_{33} + \gamma_{33A}D_A)m^2 + (\gamma_{44} + \gamma_{44A}D_A)Q^2 + (\beta_T + \beta_{TA}D_A)T^2 + (\gamma_{12} + \gamma_{12A}D_A)wr + \\
 & + (\gamma_{13} + \gamma_{13A}D_A)wm + (\gamma_{23} + \gamma_{23A}D_A)rm + (\gamma_{14} + \gamma_{14A}D_A)wQ + \\
 & + (\gamma_{24} + \gamma_{24A}D_A)rQ + (\gamma_{34} + \gamma_{34A}D_A)mQ + (\beta_1 + \beta_{1A}D_A)wT + \\
 & + (\beta_2 + \beta_{2A}D_A)rT + (\beta_3 + \beta_{3A}D_A)mT + (\beta_Q + \beta_{QA}D_A)QT.
 \end{aligned}$$

The firm-specific models of equation (37) are estimated first. The next two sections discuss the results for Firm A and Firm B, respectively.

¹⁴In order to preserve the non-singularity of the variance-covariance matrix, one equation – here the material cost share – was omitted. Under the assumption of profit maximisation, revenue share equations would also appear in the system of simultaneous equations.

ESTIMATION RESULTS FOR FIRM A

Annual observations are available for both companies, but for Firm A we had to use relatively short, 15-year time series because the company's productivity reports covered only a decade and a half. In line with our expectations and with the results of studies performed by other analysts before us, the estimation of equation (37) failed due to insufficient degrees of freedom. For the most general form, the unconstrained equation (37), we obtained non-significant estimates for several necessarily non-zero parameters, while the estimates were unreasonably high for some of the basic economic characteristics. A lengthy exploration of the various constraining null-hypotheses, however, provided some useful results. A likelihood ratio tests showed that three hypotheses, namely $\gamma_{44} = \beta_Q = \beta_T = 0$, could not be rejected. Some estimation problems (incorrect curvatures and signs) remained, however, even after introducing the constraints. Our further analyses revealed that the input structure of Firm A was quite stable throughout the period. This phenomenon suggested that the observed small changes in the input structure were probably a consequence of the changes in input prices, *i.e.*, the production technology was homothetic and the technological changes were input-neutral. These conclusions led to the testing of hypotheses $\beta_j = 0$ and $\gamma_{j4} = 0$, which gave noteworthy results. Although the hypotheses were rejected, their inclusion resulted in a statistically acceptable and economically meaningful model, which offered useful information for the specification of the shared A-B model. The following parameter estimates were obtained (*t*-values are given in brackets underneath the coefficients):

$$\begin{aligned}
 C - m = & 0.87 - 0.081D_s + 0.335(w-m) + 0.530(r-m) + 0.615Q - 0.141T + & (39) \\
 & (10.9) \quad (-3.8) \quad (94) \quad (178) \quad (22) \quad (-1.15) \\
 & + 0.144(1/2w^2+1/2m^2-wm) - 0.076(wr-wm-rm+m^2) + 0.103(1/2r^2+1/2m^2-rm), \\
 & (1.99) \quad \quad \quad (-2.64) \quad \quad \quad (3.9)
 \end{aligned}$$

where D_s is a binary dummy variable showing the effect of major structural changes in one of the years of the observation period.

Every variable is logarithmically transformed in our model because hypotheses on the linear and Box-Cox transformations were rejected for every variable. We furthermore used the usual parameter constraints ensuring input price homogeneity of degree 1 of the cost function because our test results did not allow us to reject the hypothesis.

The first-order technological parameter was non-significant. This result surfaced in most models for Firm A, suggesting that technological changes did not have a substantial direct (immediate) cost-saving effect, that is, they did not play a recognisable role in explaining changes in the company's inputs, costs and productivity. The degree of economies of scale was high and constant. The output elasticity of cost was estimated at $\varepsilon_{CQ} = 0.61$, and thus the derived input elasticity of output,

the scale elasticity was $\varepsilon_{QX} = 1.63$. The input price elasticities of cost showed little change over time. Production costs were most sensitive to capital prices and least sensitive to material prices. Demand for all three inputs was price-inelastic. For the relationships between inputs, the estimates suggested complementary between labour and material, while labour-capital as well as capital-material substitution was indicated. As this brief summary of results shows, the cost function satisfied the theoretical requirements of economic rationality, and contained no unreasonable economic properties. Various further null-hypotheses were applied to the model, but all of them were rejected. The model as shown in equation (39) can therefore be regarded as the final outcome of our explorations.

ESTIMATION RESULTS FOR FIRM *B*

The time series provided by the productivity report of Firm *B* go back all the way to the end of World War II. The oldest data had to be disregarded, however, because the technology used during the early years was fundamentally different from the one in use during the 15-year period we considered for Firm *A*. First we determined where to cut the time series. An investigation into past technological changes revealed that in one rather short time period, technological changes of so fundamental a nature occurred that it seemed reasonable to separate the long time series into periods of “old” and “new” technologies. Several cost models were estimated for various periods of observation. Based on test results, years of the “old” technologies were cut out. After discarding the observations of the years that preceded the great technology change, we still managed to lengthen the 15-year period shared with Firm *A* by 12 more years. The cost functions were then estimated with both the 27-year and the 15-year-long data sample. The parameter estimates and the economic characteristics gained from them differed only to a negligible extent but the estimates of the longer period were more efficient in a statistical sense and we decided to work with them. Tests conducted to determine variable transformations in all specifications, including the unconstrained translog cost function shown in equation (37) yielded the following final conclusions for Firm *B*:

- Linear transformation was rejected for every variable.
- Logarithmic transformation could not be rejected for the output variable and the technology index.
- Box-Cox transformation was applied to all other variables. Variable transformation parameters were obtained in the $0 < \lambda < 1$ interval.
- The hypothesis of a homothetic production process could not be rejected.

Estimation results:

$$\begin{aligned}
 C - m = & 0.009 + 0.297(w - m) + 0.542(r - m) + 0.576Q - 0.588T + & (40) \\
 & (2.26) \quad (145) \quad (253) \quad (26) \quad -4.1) \\
 & + 0.130(1/2w^2 + 1/2m^2 - wm) - 0.16(wr - wm - rm + m^2) + \\
 & (11) \quad (-14) \\
 & + 0.241(1/2r^2 + 1/2m^2 - rm) - 0.302(w - m)T + 0.34(r - m)T + \\
 & (14) \quad (-20) \quad (22) \\
 & + 0.141(1/2Q^2) + 1.43(1/2T^2) - 0.734QT \\
 & (2.11) \quad (2.45) \quad (-2.94)
 \end{aligned}$$

The statistical properties of the cost function satisfy the requirements set by economic theory and reasonable expectations. The economic characteristics are rational and reasonable. The annual estimates of scale elasticity are generally high. They increased slightly at the beginning of the period then stayed at the elevated level for several years, after which a slight decline was observed, and finally their value remained constant for the last few years of the period. During the last ten years, the scale elasticity of A was slightly higher than the scale elasticity of B but the difference is not statistically significant.

The technology elasticity of cost was negative. The estimated annual values appeared reasonable and corresponded roughly to results from engineering type investigations. Another interesting result that also matches engineering type estimates is that in the last two thirds of the period, technological changes (involving primarily the digitisation and computerisation of increasingly network-based production processes) increased the technology elasticity of cost. In other words, the direct cost-saving effect of technological changes seems to have increased.

The estimates of input price elasticities of cost were essentially the same as those obtained for Firm A for all three inputs. Once again, production costs were most sensitive to capital prices and least sensitive to material prices. As in Firm A , demand for all three inputs was insensitive to input prices; the price elasticity of capital input was somewhat lower here than in Firm A , the price elasticity of material somewhat higher, while the price elasticity of labour was the same. Inputs were shown as substitutes to each other, except in the second part of the period, when labour and capital were complementary. Null-hypotheses of numerous further possible relationships were tested but we always rejected the constraints they implied. The model described in equation (40) can therefore be considered to be the final outcome of our exploration.

The cost models estimated separately for the two firms show some profound technological similarities. This warrants the testing of *common technological* hypotheses in estimated *shared cost function*.

ESTIMATION RESULTS FROM THE SHARED ($A + B$) MODEL

For each parameter, we tested the null hypothesis that there is no difference between the two firms, *i.e.*, that the parameter estimate of the binary variable D_A does not significantly differ from zero. It was obvious, however, that not even the increased number of observation points of the shared cost function can provide sufficient degrees of freedom for the simultaneous estimation of the large number of parameters that appear in equation (38) even in the simplified case where the cost function is restricted to being homogeneous of degree 1 in input prices. Testing the equality of parameters for A and B required a large number of hypotheses which had to be tested in several steps.

Four constraints offered themselves as a point of departure. First, since the cost share of capital input was the same for the two firms in the year around which the Taylor-series expansion of the function was done, it seemed sensible to use the constraint $\alpha_{2A} = 0$, *i.e.*, to test the hypothesis that the two companies had the same first-order capital parameters. Second, as the parameters of the cost functions estimated separately suggested that the first-order output parameters were also equal, we also tested the constraint $\alpha_{4A} = 0$. The results of the cost functions estimated separately for A and B suggested the third and fourth constraints: $\gamma_{14A} = \gamma_{24A} = 0$. The constraints were applied individually and also in combination. The results were discouraging. Some parameters proved to be of poor statistical quality and unacceptable from an economic point of view (incorrect sign and unreasonable magnitude) indicating that the specification was far from being able to provide a reasonable representation of the firms' technologies.

An examination of the results revealed that most of the estimation problems were rooted in three parameters (γ_{4A} , β_Q and β_T). The realisation of this led to a re-assessment and some correction of the output and technology data and prompted an investigation of the behaviour of the three parameters in the presence of various constraints. We returned to the results of the firm-specific models once again and established that none of the three parameters was significantly different from zero for Firm A . After introducing constraints in the forms of $\gamma_{44A} = -\gamma_{4A}$, $\beta_{QA} = -\beta_Q$ and $\beta_{TA} = -\beta_T$, there was a dramatic improvement in the estimates.¹⁵

Further constraints could also be implemented because the second-order parameters were not statistically significant allowing us to introduce zero-constraints. Indeed, the hypotheses of equality between the corresponding second-order parameters of the two firms $\gamma_{11A} = \gamma_{12A} = \gamma_{22A} = 0$ could not be rejected based on the

¹⁵ The results of the tests were somewhat contradictory because the likelihood ratio test indicated that the hypotheses could be rejected while the t -statistic suggested that they could not. However, the results improved to such an extent that we elected to retain the hypotheses and use the constrained cost function as the starting point for further investigation.

likelihood ratio test. When the constraints were applied, all of the parameters of the cost function became highly significant.

The results also indicated that even more constraints could be introduced. Since the second-order parameter estimates of the input-technology interaction were close to zero for Firm A, the hypotheses $\beta_{1A} = \beta_1$ and $\beta_{2A} = \beta_2$ were tested. Neither could be rejected. At this point, we reached the limit of constraining the cost function. All further constraints were rejected. The model shown in equation (41) can therefore be considered to be the final outcome of our exploration of the single-output shared cost function

$$\begin{aligned}
 C - m = & 0.004 - 1.899D_A - 0.082D_s + (0.296 + 0.029D_A)(w-m) + & (41) \\
 & (0.74) \quad (-249) \quad (-5.4) \quad (149) \quad (10.8) \\
 & + 0.542(r-m) + 0.600Q - (0.650 + 0.556D_A)T + \\
 & (360) \quad (32.6) \quad (-4.47) \quad (5.33) \\
 & + 0.122(1/2w^2 + 1/2m^2 - wm) - 0.129(wr - wm - rm + m^2) + \\
 & (8.9) \quad (-10.1) \\
 & + 0.214(1/2r^2 + 1/2m^2 - rm) - (0.325 - 0.325D_A)(w-m)T + \\
 & (12.5) \quad (-16.7) \\
 & + (0.373 - 0.373D_A)(r-m)T + 0.010(w-m)Q - 0.022(r-m)Q + \\
 & (20.9) \quad (2.44) \quad (-5.83) \\
 & + (0.300 - 0.300D_A) 1/2Q^2 - (1.153 - 1.153D_A)QT + (2.566 - 2.566D_A) 1/2T^2. \\
 & (2.67) \quad (-3.04) \quad (2.36)
 \end{aligned}$$

where all variables, with the exception of the dummies, appear in a logarithmically transformed form.

Equation (41) describes the greatest possible extent of similarities between the production technologies of the two companies. We built these similarities into the shared cost function in a statistically and economically justifiable way and learned a great deal more from the shared models than what we knew having estimated only single-firms models. The structural information thus gained can be used to complete the decomposition and forecasting of changes in the two companies' input volumes and productivity. Before doing that, let us sum up what we have learned!

For both companies and for each year, the cost function satisfies the behavioural requirements that neoclassical production theory poses for production costs. Reasonable estimates were obtained for input demand and the relationships between the three input categories. The own-price elasticities of input demand have *a priori* correct negative signs. Demand for each of the three inputs is inelastic with respect to its own price. The inputs substitute for each other with the exception of complementary relationship between capital input and material input in Firm A.

There is a high degree of economies of scale for both companies. For Firm A, the annual estimates vary within the narrow range of $\varepsilon_{QX} = 1.65 - 1.67$. The reasonableness of this estimate can be tested by looking at the relationship between the annual output growth rates and the annual productivity growth rates. As can be

seen in *Figure A6* in the *Appendix*, this relationship is relatively stable for Firm *A*; *i.e.*, the scale elasticity is approximately constant. For Firm *B*, the annual estimates of scale elasticity follow the same characteristic path as the estimates derived from the firm-specific model and available previous estimates by other analysts. The relatively stable degree of economies of scale is explained for both companies by the opposite forces of the scale-economies-exhausting effects of output growth on the one hand, and the scale-economies-increasing effects of technological changes on the other hand. It seems that the two forces roughly counterbalanced each other. An increase in production normally reduces the degree of economies of scale if technology remains unchanged.¹⁶ However, those technological changes whose cost saving effects expand as the volume of output increases lead to increases in the degree of economies of scale. The rapid growth and the rapid technological progress appeared to be more or less in balance during the observation period for Firm *A*. For Firm *B*, however, we may argue that the effects of rapid technological progress exceeded the effects of the gradually decelerating growth of the firm during the observation period.

The annual estimates of the technology elasticity of cost are not completely satisfactory. For Firm *A*, the annual estimates remain constant over time at the value of $\varepsilon_{CT} = -1$. This is considered reasonable. For Firm *B*, however, the absolute values of the *a priori* correctly negative estimates are slightly higher than what we could accept as reasonable. Finally, when the annual estimates are based on the longer 27-year sample period, they show a trend. A weaker impact on costs during the first 12 years is followed by a temporally increasing trend. If, however, the model overestimates the technology elasticity of costs, it will also underestimate its counterpoint, the degree of economies of scale. A comparison with the results of the firm-specific model indeed appears to support the suspicion that the shared model somewhat underestimated the scale elasticity of Firm *B* (while somewhat overestimating it for the years preceding the shared period.)

The shared cost function described in equation (38) and the estimation results shown in equation (41) rely on the assumption that the products of both companies can be aggregated into a single output. The single-output model gave statistically valid and economically reasonable results, which are often perfectly suitable for cost analysis and the decomposition of temporal changes in productivity and of inter-firm differences in those temporal changes. But there are two hidden dangers in using single-output models. First, the estimates may be biased if the output aggregate does not exist. Second, information with respect to individual output categories may be needed for both regulatory and management purposes. Impacts

¹⁶ This phenomenon tends to exist for established firms that have been operating for a long time. With new or young firms, however, the opposite phenomenon may also occur, *i.e.*, an increase in the size of production may be accompanied by an increase in the degree of economies of scale. Our firms *A* and *B* are old, established, large enterprises.

by the output aggregate need to be decomposed into individual output effects. This can be achieved by estimating *multi-output* shared cost functions. A multi-output model offers valuable information: its estimates show the roles of individual outputs in productivity growth, and reveal similarities and differences in the interactions between technological changes and individual outputs. When faced with output-augmenting technological progress, it is an especially useful feature that we can assign output-specific effects.

Both companies have three main output categories (α , β and γ , see *Table A1*, where their temporal proportional changes are shown). With these inserted into equation (38), the number of first- and second-order parameters waiting to be estimated increases to such an extent that it exceeds the number of observations, therefore the parameters of the three-output model cannot be estimated. In our case the number of observations is insufficient to gain efficient estimates even for two-output models. Decomposition of productivity growth rates and inter-firm differences in productivity growth rates are accomplished using the single-output model. To overcome the difficulty caused by the insufficient number of observations, it is advisable to base the productivity measurement and analysis on at least quarterly, and preferably on monthly, data.

DECOMPOSITION OF PRODUCTIVITY GAINS

During the 13-year observation period,¹⁷ the average annual productivity gain (proportional change in productivity) in Firm *A* was 2.52 percentage points higher than the corresponding rate in Firm *B*. We now attempt to find out why. The inter-firm difference is decomposed into several causal components displayed in *Table 3*. The component which is generated by economies of scale can be further decomposed according to the following formula:

$$\dot{E}_A - \dot{E}_B = \frac{(\dot{Q}_A + \dot{Q}_B)}{2} (\xi_{cQ_A} - \xi_{cQ_B}) + \frac{(\xi_{cQ_A} + \xi_{cQ_B})}{2} (\dot{Q}_A - \dot{Q}_B). \quad (42)$$

The firm-specific and the shared models show strong similarities with respect to our most important empirical result: the fairly large – 2.52 percentage point – difference between the two firms' productivity growth rates was almost entirely (in 95-96 percent) the consequence of Firm *A* growing more rapidly than Firm *B*. The remaining effects were individually negligibly small, even if combined. As they showed very small values, we did not deem it necessary to further decompose the technology effects.

¹⁷ For a period of 13 years a total of 12 growth rates can be computed, since growth in the first year is not known.

TABLE 3 • The decomposition of average proportional productivity change

Effects		Firm A		Firm B		A + B	
		Rate	Percent distribution	Rate	Percent distribution	Rate	Percent distribution
<i>Firm-specific models</i>							
Productivity growth	$\hat{\phi}$	6.34	100	3.82	100	2.52	100
Technological effect	\hat{B}	0.69	11	0.75	20	-0.07	-3
Growth effect	\hat{E}	5.45	86	3.49	91	1.98	79
Due to economies of scale	$\overline{Q\Delta\xi}$	-	-	-	-	-0.45	-17
Due to output increase	$\overline{\xi\Delta Q}$	-	-	-	-	2.43	96
Residual effect	\hat{R}	0.20	3	-0.42	-11	0.61	24
<i>Shared model</i>							
Productivity growth	$\hat{\phi}$	6.34	100	3.82	100	2.52	100
Technological effect	\hat{B}	0.45	7	0.84	22	-0.39	-15
Growth effect	\hat{E}	5.66	89	3.30	86	2.36	94
Due to economies of scale	$\overline{Q\Delta\xi}$	-	-	-	-	-0.04	-1
Due to output increase	$\overline{\xi\Delta Q}$	-	-	-	-	2.40	95
Residual effect	\hat{R}	0.23	4	-0.32	-8	0.55	22

In Firm A, 86–89 percent of productivity growth is due to the rapid increase in the volume of the firm's output in the presence of substantial degrees of economies of scale. The direct cost saving due to technological changes is responsible for only 7–11 percent of the actual productivity gain. We mentioned the possibility of such a result when we showed that there was a strong correlation between the annual growth rates of Firm A's output and productivity, while the technological changes correlated rather weakly with changes in productivity during the observation period. We then surmised that the main reason for the introduction of the new technologies was probably not the immediate and short-term cost saving. We later added that technological progress was more likely driven by the expected positive effects of new technologies on economies of scale. It was expected that cost savings due to the introduction of new technologies would gradually emerge and increase over time as the volume of output increased over a longer period of time.

The estimates seem to support this reasoning. It is an interesting result that the technology of Firm A had a weaker effect on productivity growth (*i.e.*, caused a smaller immediate cost reduction) than the technology of Firm B even though technological progress was faster in A than in B. The explanation may be that Firm A's markets, and hence its output and revenues grew at a very high rate. Fast output growth forced – and rapidly increasing revenues allowed – the introduction of new technologies at an ambitious pace, which – precisely because of its ambitious nature – resulted in extra costs and thus curbed the extent of immediate and short-term cost savings.

Turning to Firm *B*, 86–91 percent of productivity growth is due to the increase in output in the presence of economies of scale. The immediate and short-term cost-saving effects of technological changes are responsible for 21–22 percent of the average productivity gain. This finding is consistent with previous estimates available from the company.

The residual effect left unexplained by the model is fairly high, especially for Firm *B*, where it represents 8–11 percent of productivity growth. This phenomenon is due to the high variability of annual productivity gains. In a small sample – a 13-year period in our case – highly variable individual residuals can have a strong effect on the mean residual. High temporal variability of productivity gains is a widely observed phenomenon. Corporate reports show that productivity growth – with, it is safe to claim, few exceptions – tends to proceed at an uneven pace over time. There are several reasons for this. An especially important reason is that temporarily unused capacities are necessarily created during investments, since capital input exhibits high degrees of indivisibility. Unused capacities temporarily decrease the annual productivity growth rate, and when the unused capacities are finally utilised, their presence accelerates the annual growth in productivity.

DECOMPOSITION OF INPUT GROWTH RATES

Input growth rates are one of the two components of productivity gains. *Table 4* offers their decomposition in firm-specific and combined *A + B* models. Average annual growth rates are broken down to a technology effect, an output growth effect, and as many input price effects as the number of inputs. We work with three input categories: labour, capital and material. Output growth exerts the most important effect on the volumes of inputs. In fact, its effect is so important that it exceeds that of the growth rate in productivity. This is possible only if the combined other effects are negative in the sense that they make input volumes to decline. As can be seen in *Table 4*, this is what happens for all three inputs.¹⁸ This is not surprising, given that the majority of input prices increased and the majority of technological changes had an input-reducing effect. The input-saving effect of technological changes shows substantial annual fluctuation. For Firm *B*, there was a year when the annual capital-saving effect was as low as zero, but in another year a saving as high as 2.19 percentage points was achieved. The latter is a quite exceptional figure, but our examination of the events of that year convinced us that it was a valid estimate.

¹⁸ For all inputs in Firm *A* and for labour input in Firm *B*.

TABLE 4 • Decomposition of the average growth rates of inputs

Effects		Firm A		Firm B		A + B	
		Rate	Percent distribution	Rate	Percent distribution	Rate	Percent distribution
Labour input	\dot{L}	8.73	100	2.69	100	6.04	100
Technological effect	\dot{B}_L	-0.45	-5	-2.19	-81	1.74	29
Growth effect	\dot{E}_L	8.93	102	5.16	192	3.77	62
Labour price effect	\dot{w}_L	-2.28	-26	-3.16	-117	0.88	15
Capital price effect	\dot{r}_L	1.36	16	0.73	27	0.63	10
Material price effect	\dot{m}_L	1.22	14	1.28	48	-0.06	-1
Residual effect	\dot{R}_L	-0.05	-1	0.87	32	-0.92	-15
Capital Input	\dot{K}	7.30	100	4.69	100	2.61	100
Technological effect	\dot{B}_K	-0.45	-6	0.00	0	-0.45	-17
Growth effect	\dot{E}_K	7.93	109	4.57	97	3.36	129
Labour price effect	\dot{w}_K	0.69	9	0.64	14	0.05	2
Capital price effect	\dot{r}_K	-0.62	-8	-0.44	-9	-0.18	-7
Material price effect	\dot{m}_K	-0.19	-3	0.03	1	-0.22	-8
Residual effect	\dot{R}_K	-0.06	-1	-0.11	-2	0.05	2
Material input	\dot{M}	8.01	100	6.31	100	1.70	100
Technological effect	\dot{B}_M	-0.45	-6	-1.20	-19	0.75	44
Growth effect	\dot{E}_M	9.86	123	5.55	88	4.31	254
Labour price effect	\dot{w}_M	2.92	36	3.68	58	-0.76	-45
Capital price effect	\dot{r}_M	-0.92	-11	0.09	1	-1.01	-59
Material price effect	\dot{m}_M	-2.25	-28	-2.44	-39	0.19	11
Residual effect	\dot{R}_M	-1.15	-14	0.63	10	-1.78	-105

Increases in input prices substantially reduced the growth rates of inputs, especially those of labour, because labour was the most price sensitive input. Increases in labour prices generated an average annual decline of 2.28 percentage points in the use of labour in Firm A, and 3.16 percentage points in Firm B. Material input also showed high degrees of price sensitivity: 2.25 and 2.44 percentage points annually on average for Firms A and B, respectively. Cross-price elasticities tended to be smaller than own-price elasticities. One exception was the price of labour input, which had a substantial effect on the use of material inputs in both companies.

Averaging the annual rates over the 13-year observation period, Firm A increased the volume of labour input 6 percent faster, capital input 2.6 percent faster and material input 1.7 percent faster than Firm B. The differences between input growth rates are due mainly to the faster increase of output volumes in Firm A than in Firm B. Relative to Firm B, the input-saving effect of technological progress in Firm A accelerated the increase in labour and material inputs but decelerated the growth of capital.

Inter-firm differences in input prices had only a mild effect on the differences in input growth rates in the case of labour, an especially mild effect with respect to capital, but an outstandingly strong effect on material. Owing to faster increases in input prices and higher price elasticities of demand for inputs, overall the input price effect caused the average annual growth rate of productivity of Firm A to be 1.5 percentage points lower than the corresponding indicator of Firm B.

The unexplained residual effect is very high for material input. This is caused mostly by the high annual fluctuation in the material input of Firm A that is also apparent in *Table A2*. In contrast, since the capital input increased relatively steadily over time for both companies, the residual effect is negligibly low for capital. Five causal factors are shown in *Table 4*. They could be further decomposed into *size* effects and *intensity* effects. Such decomposition, however, would fall beyond the scope of the present study.

CONCLUSION

As noted in the introduction, doing everything in their power to facilitate the efficient operation of the firms they regulate is one of the most important duties of socially responsible regulators. They cannot carry out their duty, unless they understand productivity. They must measure, compare and analyse corporate productivity in various ways, using an arsenal of economic and econometric analytical tools.

Historically, productivity studies were developed first by regulated monopolies for their own use as well as for regulatory purposes. Beginning in the 1960's, regulators of monopolies made extensive use of them for several decades, especially following the world-wide spread of price cap regulation. However, with the advent of the competitive era; *i.e.*, the introduction of competition into formerly monopoly markets, productivity studies were forced into the background by not more important but more urgent problems of imperfectly competitive markets. However, it is easy to see the reasons why productivity is even more important for competitive companies and regulators of imperfectly competitive markets than it used to be for monopolies and their regulators.

Productivity analysis is also an important and useful tool in the hands of corporate management. The competitiveness and market position of regulated as well as unregulated companies, the price and quality of their products and services, and ultimately their profitability all depend on how rapidly they may be able to improve their productivity. They also must study and understand productivity.

From a corporate as well as a regulatory point of view, a renaissance of productivity studies in the not so distant future would be very much in order. This paper is a small contribution showing how certain useful productivity analyses, particularly those that involve inter-firm comparisons and causal decompositions, could be conducted for management and regulatory purposes.

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APPENDIX

TABLE A1 • Annual proportional changes* in the output volumes of Firms A and B ($100 \hat{q}_{it}$)

Year	Output α			Output β			Output γ			Total output		
	A	B	A-B	A	B	A-B	A	B	A-B	A	B	A-B
1	10.8	7.5	3.4	15.2	13.9	1.3	-3.2	7.7	-10.9	12.9	9.9	3.0
2	9.2	6.7	2.5	12.7	8.5	4.2	14.6	8.2	6.3	11.7	7.5	4.1
3	9.3	6.4	2.9	10.8	4.4	6.3	7.1	7.0	0.1	10.1	5.7	4.5
4	11.2	7.5	3.7	16.5	12.1	4.4	3.6	9.8	-6.2	14.3	9.5	4.8
5	9.0	7.6	1.5	15.5	15.0	0.5	9.3	-18.4	27.7	13.3	10.0	3.3
6	14.3	8.3	6.0	21.0	12.8	8.2	11.9	12.5	-0.6	18.7	10.3	8.4
7	15.2	7.8	7.4	19.0	13.0	6.0	-0.9	18.9	-19.7	17.2	10.4	6.8
8	13.9	6.0	7.9	7.9	8.5	-0.6	8.5	18.6	-10.1	9.6	7.5	2.1
9	11.3	5.1	6.2	11.6	8.3	3.4	10.3	12.5	-2.3	11.5	6.7	4.7
10	12.7	4.2	8.4	17.4	11.3	6.1	14.7	29.2	-14.5	15.8	8.3	7.6
11	15.1	3.2	11.9	18.6	8.3	10.3	10.0	10.5	-0.4	17.2	5.8	11.4
12	17.2	4.9	12.3	19.0	10.4	8.7	19.2	13.6	5.5	18.5	7.9	10.7
13	16.1	3.4	12.7	12.5	10.7	1.8	18.1	11.4	6.8	13.8	7.2	6.6
Mean	12.7	6.0	6.7	15.2	10.5	4.7	9.5	10.9	-1.4	14.2	8.2	6.0

* The proportional changes in output are defined by equation (3).

TABLE A2 • Annual proportional changes* in the input volumes of Firms A and B ($100 \hat{x}_{it}$)

Year	Labour			Capital			Materials			Total input		
	A	B	A-B	A	B	A-B	A	B	A-B	A	B	A-B
1	2.4	1.7	0.7	6.7	5.7	1.0	-5.0	17.5	-22.5	3.5	6.4	-2.9
2	13.1	2.1	11.0	6.1	4.9	1.2	2.4	0.0	2.4	8.0	3.3	4.6
3	8.0	-1.0	8.9	6.6	4.8	1.8	2.6	17.2	-14.6	6.5	5.2	1.3
4	2.4	0.1	2.3	6.8	5.5	1.3	9.0	2.3	6.7	5.5	3.5	2.1
5	10.7	4.6	6.1	4.5	4.7	-0.2	4.2	5.7	-1.5	6.6	4.9	1.7
6	8.3	4.1	4.2	3.5	4.8	-1.4	10.8	5.2	5.6	6.0	4.7	1.4
7	21.8	-2.7	24.4	10.7	5.8	5.0	18.0	-2.5	20.5	15.6	1.9	13.7
8	3.2	4.0	-0.8	14.1	5.2	8.8	19.7	7.1	12.6	11.0	5.1	5.9
9	11.6	4.1	7.5	8.6	5.5	3.1	-5.9	8.8	-14.7	7.7	5.6	2.1
10	5.0	7.4	-2.4	5.3	3.8	1.5	20.7	9.2	11.4	7.2	5.9	1.3
11	3.7	2.6	1.1	6.9	2.7	4.2	18.2	1.0	17.2	7.5	2.4	5.2
12	17.6	1.8	15.8	7.4	3.7	3.7	-3.2	8.6	-11.8	9.0	3.9	5.1
13	5.7	6.1	-0.4	7.8	3.8	4.0	12.9	2.0	10.9	7.9	4.3	3.6
Mean	8.7	2.7	6.0	7.3	4.7	2.6	8.0	6.3	1.7	7.8	4.4	3.5

* The proportional changes in input are defined by equation (6).

TABLE A3 • Indices of technological change for Firms A and B

Year	Year 0 = 1,0		Year 5 = 1,0		Previous year = 1,0		100 T _t ⁺	
	A	B	A	B	A	B	A	B
0	1.00	1.00	0.37	0.58	–	–	–	–
1	1.05	1.12	0.39	0.64	1.05	1.12	4.88	11.10
2	1.27	1.27	0.47	0.73	1.21	1.13	19.20	12.55
3	1.31	1.37	0.48	0.79	1.03	1.08	2.59	7.83
4	2.23	1.54	0.83	0.89	1.71	1.13	53.44	11.98
5	2.69	1.74	1.00	1.00	1.21	1.12	19.02	11.73
6	2.97	2.07	1.10	1.19	1.10	1.19	9.62	17.44
7	3.94	2.34	1.46	1.35	1.33	1.13	28.49	12.29
8	4.94	2.67	1.83	1.54	1.25	1.14	22.48	13.24
9	6.90	2.94	2.56	1.69	1.40	1.10	33.44	9.77
10	8.12	3.16	3.01	1.82	1.18	1.07	16.31	7.23
11	9.47	3.43	3.51	1.98	1.17	1.09	15.32	8.20
12	10.04	3.55	3.73	2.04	1.06	1.03	5.92	3.26
13	13.24	4.17	4.92	2.40	1.32	1.17	27.66	16.09

FIGURE A1 • Annual proportional changes in output in Firms A and B

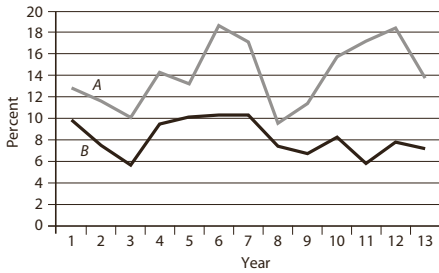


FIGURE A3 • Annual proportional changes in output and input in Firm A

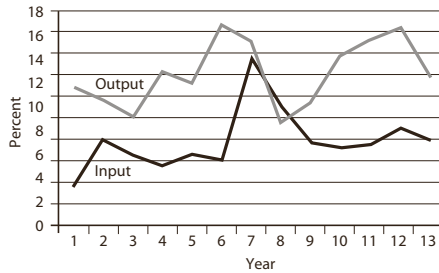


FIGURE A2 • Annual proportional changes in input in Firms A and B

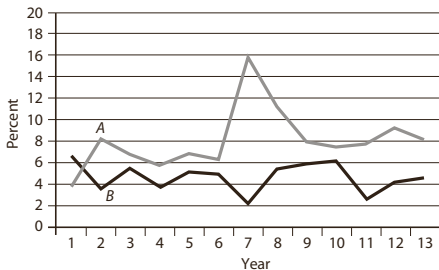


FIGURE A4 • Annual proportional changes in output and input in Firm B

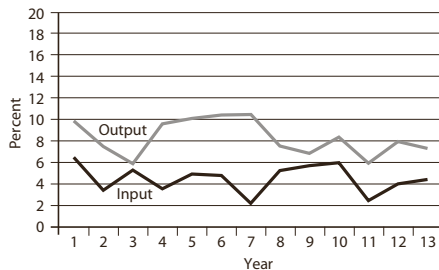


FIGURE A5 • Annual productivity gains in Firms A and B

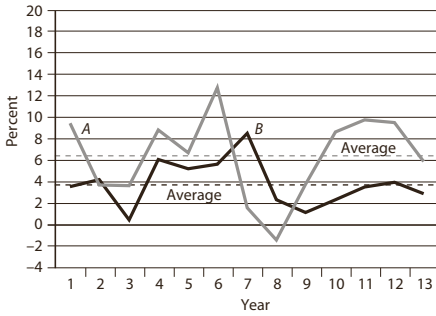


FIGURE F8 • Annual changes in the technology index and productivity of Firm A

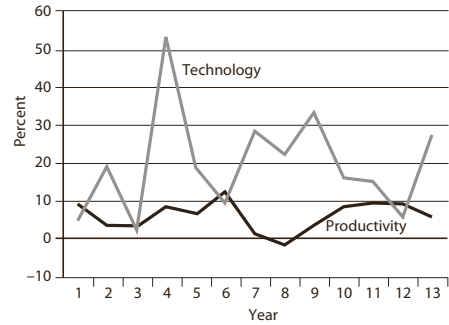


FIGURE A6 • Annual changes in the output and productivity of Firm A

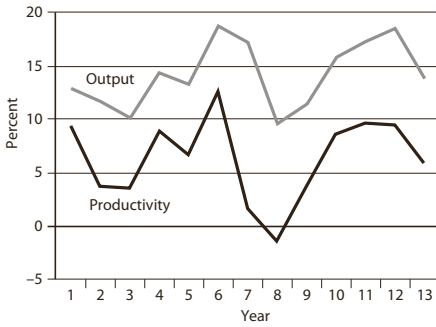


FIGURE 9 • Annual changes in the technology index and productivity of Firm B

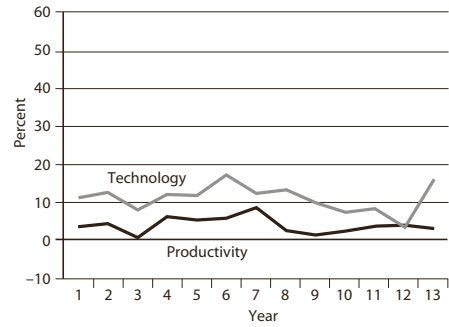


FIGURE F7 • Annual changes in the output and productivity of Firm B

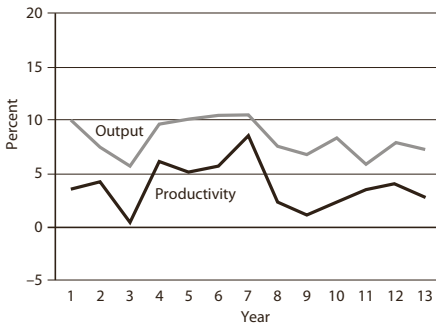


FIGURE F10 • The technology indices of Firms A and B

